Credit Constraints and Productivity of SMEs: evidence from Canada*

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Abstract

To what extent firms are constrained by external credit is usually unobserved in commonly used firm-level data. We use a survey of financing among Canadian small and medium-sized enterprises to measure the likelihood of a firm being constrained by credit. We find that firm size, current debt-to-asset ratio and cash flow are robust indicators of being financially constrained, while long-term debt to asset ratio is not a significant indicator of credit constraints. We then estimate the firm-level total factor productivity, taking into account the measured credit constraints. Omitting credit constraints leads to an upward bias of productivity estimates, by 4 percent. In addition, we find no strong evidence that suggests credit constraints lead to slower productivity growth. Finally, we confirm that both investment and employment growth are negatively affected by the measured credit constraints.

Keyword: productivity, financial constraint, firm growth.

JEL codes: D24, G32, L25

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1 Introduction

The extent to which external financial constraints due to frictions in credit markets contributes to misallocation and reduces aggregate productivity has been widely studied and is an issue with important policy implications.[1] Despite its importance and the voluminous research, existing studies often rely on commonly used data of large firms (e.g., Compustat), credit constraints facing small and medium-sized business are still not well understood. Researchers in designing their study sometimes split samples according to firm size, implicitly speculating that small firms may be more likely constrained by external credit than large firms.[2]

Is the pre-assumption warranted that small firms are uniformly more likely constrained by external financing than large firms? To what extent observed investment and employment are driven by productivity versus credit constraints? In this paper, we study the linkage between credit constraints and productivity using data on small and medium-sized enterprises (SMEs) in Canada. We first measure and estimate the degree to which small and medium-sized firms are financially constrained, by exploiting detailed information on financing activities and outcomes. We then estimate the firm-level productivity by taking into account the measured degree of financial constraint, overcoming a bias of productivity estimates if credit constraints are omitted from the estimation.

We measure the likelihood of a business being constrained by credit in a similar way as in Kaplan and Zingales (1997), but our measure is constructed solely from the observed outcome of external financing. Thus, our measure more accurately reflects the degree of credit constraints than the Kaplan-Zingales index. As pointed out in Hennessy and Whited (2007), the Kaplan-Zingales index of financial constraints measures the firm's "need" for external financing, rather than the degree of being constrained due to credit market frictions. In this paper, the data allow us to separate financial constraints from the need for external financing.

Associating the above likelihood with firm characteristics, our estimation shows that the current debt-to-asset ratio and cash flow are robust indicators that can be used to proxy for financial constraints among SMEs. Long-term debt to asset ratio is, however, statistically insignificant as an indicator of financial constraints. Firms with a higher cash flow are both less likely to need external financing and less likely to be constrained if they do need it. We also find that firms with a smaller

[1] In this paper, we use interchangeably external financial constraints and credit constraints, they both refer to external lending as the observed equity financing is tiny among firms in our data.

[2] See for example, Fazzari, Hubbard, and Peterson (1988), Gertler and Gilchrist (1994), and Gilchrist and Himmelberg (1995), among many others.

asset size are more likely constrained, but the firm size in indicating being constrained by credit is less important than cash flow and current debt.

In estimating the firm-level productivity, coefficient estimates for production function can be biased if financial constraints are omitted from the estimation, because data show that both investment and employment are negatively correlated with the measured financial constraints. Our results show that this negative correlation leads to a downward bias of coefficient estimates for capital and labor, if credit constraints and external financing are omitted from the structural estimation of production function. The resulted estimates of total factor productivity in turn are biased upwards, by 4 percent. Additionally, our estimation suggests that measured financial constraints and the estimated total factor productivity are negatively correlated, even after we overcome the omission bias.

This paper makes two contributions to better understanding financial constraints and their real effects. First, our paper adds to existing studies on credit constraints, particularly among small firms. Whether firms are constrained by credit is mostly not observed or measured in many data sets. Empirical studies using firm-level data usually rely on financial conditions (e.g., debt-to-asset ratio and cash flow) and firm characteristics (e.g., size and age) to indirectly approximate the likelihood of a firm being financially constrained. These proxy variables may fail to measure the true likelihood of being financially constrained.[3] Our measure improves upon the proxy variables as it is based on observed outcome of external financing. To our best knowledge, the current paper is the first to measure financial constraints among SMEs solely from the observed outcome of external financing.

This paper also contributes to understanding the financial constraint-productivity nexus at the firm level, based on productivity estimation where we take into account financial constraints. Accounting for financial constraints can correct, to some degree, an upward bias of the estimated firm-level productivity due to omitting financial constraints. In quantitative models with financial constraints and heterogeneous firms, such as Caggese (2007) and Midrigan and Xu (2014), the firm's productivity process is usually estimated without taking into account the impact of credit constraints.

Further, at the micro level, there is a lack of evidence on the financial constraint-productivity nexus. In commonly used firm-level data (e.g., Compustat), neither total factor productivity nor financial constraints are readily observed and measured. Total factor productivity is usually measured through the estimation of production function as in Olley and Pakes (1996), and unobserved financial constraints are replaced with proxy variables. If a firm's investment and employment are con-

[3] For instance, the empirical finding of the cash-flow sensitivity of investment is taken as an indirect evidence of imperfect capital markets, see Fazzari, Hubbard, and Peterson (1988) and Gilchrist and Himmelberg (1995). However, studies show that the cash-flow sensitivity of investment does not necessarily imply that firms are financially constrained, as shown by Kaplan and Zingales (1997), Cooper and Ejarque (2003), Alti (2003), and Abel and Eberly (2011), among others.

strained by external financing, a proper estimation of total factor productivity requires that financial constraints are taken into account, which in turn need to be properly measured and estimated.[4] An estimation of the production function that incorporates financial variables, e.g., debt-to-asset ratio and cash flow, does not identify the role of financial constraints, because external debt as a state variable is present in the estimation regardless of whether the model allows for financial constraints in the firm's dynamic problem.

In the rest of the paper, we review related literature in Section 2. In Section 3, we measure and estimate the degree to which firms are constrained by external financing. In Section 4, we estimate the production function by taking into account the measured financial constraints. In Section 5, we estimate to what extent investment and employment growth are affected by financial constraints and total factor productivity. Finally, we conclude the paper in Section 6.

2 Related Literature

This paper is associated with studies on estimating and quantifying the importance of financial frictions in affecting real variables, such as investment, employment, and productivity. Whether the investment sensitivity with respect to cash flow suggests binding financial constraints is not conclusive in empirical studies. See, for example, Fazzari, Hubbard, and Peterson (1988) and Kaplan and Zingales (1997) for the debate. In these studies, whether a firm is financially constrained is not directly observable in the data. Typically, the investment-to-capital ratio is estimated in regressions using measures of the average value of investment (e.g., Tobin's q), cash flow, and firm characteristics. Neither the underlying productivity process nor the degree of financial constraints is estimated in these models. Instead, financial constraints are approximated with a set of observables that are assumed to indicate the firm's (in)ability to borrow.

Cooper and Ejarque (2003) estimate a structural model of investment with market power. Their simulated model without a borrowing constraint can replicate the cash flow sensitivity of investment in the q regression by Gilchrist and Himmelberg (1995), suggesting that a strong correlation between investment and cash flow may not indicate the importance of financial frictions. Abel and Eberly (2011) and Alti (2003) also show that the investment-capital ratio is positively correlated with both Tobin's q and cash flow in environments without financial frictions. However, these studies do not

[4] Some recent studies do estimate the impact of financial conditions to productivity, for example Ding, Guariglia, and Harris (2016) and Gatti and Love (2008). In these studies, whether a firm is financially constrained is unobserved nor measured, financial variables such as cash flow to capital ratio are used as a proxy for financial constraints. These studies essentially estimate the impact of financial conditions, instead of financial constraints.

quantify to what extent financial constraints affect investment. Caggese (2007) shows that, indeed, financial frictions are theoretically important in investment dynamics. In his estimation, the average debt-to-asset ratio is again used to calibrate the parameter of collateral constraint.

Focusing on investment only (without estimating the production function), Whited and Wu (2006) estimate the Euler equation for investment with financial constraints. Since neither the non-negative dividend constraint nor the credit constraint is observed in data, the authors use observed variables as a proxy for the Lagrangian multipliers for these constraints in the Euler equation (i.e., by replacing the multiplier with a linear function of the long-term debt-to-asset ratio, cash-flow-to-asset ratio, sales growth, firm size, etc.). The drawback is that the estimation based on Euler equation may not be able to separately estimate unobserved productivity and the severity of credit constraints.

Our method of measuring credit constraints is close related to that by Kaplan and Zingales (1997), which later was extended by Hadlock and Pierce (2010). These studies use samples of firms and rank them by the extent to which they are financially constrained. Firms are classified as being financially constrained, by the authors' judgment based on their reading of firms' annual reports, filings to government regulators, and financial statements. Kaplan and Zingales (1997) find that firms classified as being less financially constrained exhibit a significantly greater cash-flow sensitivity of investment than firms that are more financially constrained. Hadlock and Pierce (2010) instead conclude that a measure based solely on firm characteristics (e.g., size and age) better reflect financial constraints. Our measure differs from that by these studies in that we do not rely on the firm's income statements and balance sheets. We label firms as being financially constrained based on observed financing activities and outcomes, this allows us to separate the "need" for external financing from the degree of being constrained conditional on needing external financing. Finally, our measure shares similarity with Levenson and Willard (2000), though these authors do not link their measured probability of credit constraints with the firm's balance sheets.

Similar to Kaplan and Zingales, Ferrando and Ruggieri (2015) also measure financial constraints based on the firm's balance sheets and income statements. Farre-Mensa and Ljungqvist (2016) show that firms labeled as "constrained" by existing measures of financial constraints on average do not differ from other firms in their ability to raise external financing.

Levine and Warusawitharana (2014) study the impact of financing on the firm's productivity growth. In their model, productivity is endogenously determined by investment in research and development. But they do not relate financial constraints to productivity within their model. Instead, they estimate the total factor productivity in reduced form or without taking into account financial constraint. They then examine how estimated productivity growth is affected by the change in debt level. The degree of financial constraints is not measured in their data. In studies that are closer to this paper, production function is estimated by taking into account financial variables (not measured financial constraints), see for example Ding, Guariglia, and Harris (2016).

An alternative measure of financial constraints used in previous research is the business credit score, assigned by commercial credit rating agencies. As surveyed in Wagner (2014) in the context of credit constraints and firm exports, these alternative scores are measured based on various information regarding a firm's liquidity condition, indebtedness, payment behavior, firm characteristics, etc. These scores are similar to Kaplan and Zingales (1997) in the sense that scores are assigned using information of balance sheets and income statements, rather than directly using the outcome of financing.

Finally, there is a long literature on information asymmetry as a form of frictions in affecting a firm's capital structure, as has been synthesized in Tirole (2006). Myers (1984) shows that the pecking order hypothesis of capital structure can be predicted by models of financing with asymmetric information. Majority of the literature along this line focuses on public corporations, while our data sample includes mostly small private businesses where the information frictions could be more severe. Though we do not have sufficient information to infer the importance of information asymmetry, our measure of financial constraints is partly based on the pecking order theory.

3 Measuring and Estimating Credit Constraints

Theoretically, a firm is credit constrained if its realized external financing is lower than the desired amount if there were no frictions. Sources of frictions can be asymmetric information on project quality, incomplete financial contracts, limited enforcement of financial contracts, and even search frictions in credit markets. This concept is similar to Kaplan and Zingales (1997), who define that a firm is financially constrained if there is a wedge of costs between internal and external funds, by which any firm is possibly constrained. These various forms of financial frictions facing small firms are not readily observed in data. Nevertheless, information on activities and outcomes concerning external financing can provide evidence on the impact of those frictions.

We use the Survey on Financing and Growth of Small and Medium Enterprises (SFSME) in Canada, and additional information from the administrative data for those surveyed firms.[5] Appendix A de-

[5] The SFSME datasets are available to researchers through a process of application. Detailed information on data access and cost can be obtained by contacting the Canadian Centre for Data Development and Economic Research (CDER) at

scribes the data in more details. The SFSME, a cross-sectional data set, reports detailed information on activities and outcomes of financing among small and medium-sized enterprises. We focus on the 2011 data because they provide richer information on financing.[6]

Information on balance sheets, income statements, and employment for firms surveyed in the SFSME is extracted from the longitudinal administrative business data. This latter information is available for the period 2008 to 2013. The advantage of using the Canadian data is that both the financing activities and the statements of income and balance sheets are available for small and medium-sized firms, allowing us to examine relations between observed credit constraints and firms' financial conditions.

Most small businesses are in sectors of construction, retail trade, professional services, accommodation, and other services. Manufacturing firms account for 7 percent of the sample. About 76 percent of businesses have fewer than 10 employees. The average firm size is 9 employees, and the median is 4 employees. The median firm age is 12 years, while both the median and mean ages of business owners are 52 years. About 70 percent of businesses were created from scratch by the owner, and the rest were acquired from others.

3.1 Measure of financial constraints

We measure the likelihood of firms being financially constrained by assigning scores to firms using their information on the activities and outcomes related to external financing. Firms with a higher score are more likely to be credit constrained. Firms in the SFSME data report whether they requested external financing and the outcomes if they did. Types of financing instruments include short-term and long-term loans, equity, trade credit, lease financing, and government assistance. Also reported are reasons for not requesting external financing, many firms reported that they did not need external financing while others saying that external financing is too costly. We want to emphasize that the measured credit constraints only speak to the observed outcome of external financing, our measure does not distinguish firms being financially distressed due to existing high debt-to-asset ratio from firms being productive but unable to obtain external financing.

We assign the highest score to firms reporting that they did not request external financing although they needed it, assuming that these firms are the most likely to be constrained by external

statcan.cder-cdre.statcan@canada.ca or visiting its website https://www.statcan.gc.ca/eng/cder/index.

[6]The 2007 data of SFSME was also available to us, which we do not use. This is because no information in the 2007 survey is available regarding the reported reasons for not requesting external financing. The lack of this information prevents us from counting firms that were constrained such that they did not request for financing.

financing. On the other end, we assign the lowest score to firms that requested external financing and obtained the requested amount for the requested types of external financing. For these firms with the lowest score, interest rates associated with external financing need to be within reasonable ranges relative to the average interest rates for the same type of financing. In between the lowest and highest scores are firms that requested financing but the outcomes were lower than requested. For example, financing requests were rejected for some firms and approved with a partial amount for others. Some firms resorted to the government for direct loans or loan guarantees, while others paid extraordinarily high interest rates relative to the majority for the same types of financing. Details on score assignment are described in Appendix B. The size difference of assign scores does not matter since we are only concerned about the relative position of firms.

We also assign scores based on the types of external financing. The pecking-order theory of financing suggests that firms first use internal funds, followed by loans, then by equity. This suggests that firms issuing new equity are more likely to be constrained than those that only requested loans. It turns out that a tiny fraction of firms requested equity financing, leading to that few firms are constrained by this criteria.

At the end, there are three categories of financial constraints: "most likely constrained", "likely constrained", and "unlikely constrained". Scores among the likely constrained firms are different from each other: we pool them together because the sample size is relatively small for this middle category. In 2011, 43 percent of small and medium-sized firms needed or requested external financing (see Table B.3). About 23 percent of all firms are likely or most likely constrained, accounting for more than half of firms that needed or requested external financing. About 9 percent of all firms were most likely constrained, accounting for 20 percent of firms that needed or requested external financing, most of these firms needed but did not request external financing reporting reasons of high financing costs. A small fraction, 2.7 percent of all firms, had their requests for external financing rejected.

In the 2011 data, 57 percent of all firms did not request external financing, citing the reason "Financing not needed". Our understanding is that these firms did not need any form of financing (internal and external). We do not assign the degree of being constrained by credit to these firms, because of the lack of information on financing, thus the measure of credit constraints for them is missing. We do not assume that firms not needing external financing is unconstrained. They may be not constrained by external credit, either because their production activity is low such that they indeed do not need financing, or because their internal funds can finance any activity. Not needing external financing may also be because firms have accumulated sufficient internal funds out of precautionary motive in previous years when they were constrained by credit. Nevertheless, being constrained in previous years does not warranty that firms are constrained in current year. Later in estimation, we apply the ordered probit model with sample selection to take into account the missing outcome for firms not needing financing.

It is likely that the above method under-measures the actual fraction of firms being credit constrained. For example, we assumed that firms are unconstrained if their requests for external financing were fully approved. These firms can be constrained, because the firm's decision on financing presumably had taken into account financial frictions. In that case, the full approval of a request can be an outcome reflecting the impact of underlying financial frictions. No direct information is available in the SFSME data to allow us to deal with this issue, firms were not asked to report reasons for requesting a particular amount of external financing. Because of this, our measure of credit constraints may underestimate the true degree to which a firm is constrained. This issue though is not unique to the current paper. For example, Tobin's q, widely used in estimating firm investment, is measured using the firm-level data (e.g., the Compustat). Tobin's q is also subject to the similar problem, because any observed firm value is already an outcome realized possibly when the firm is financially constrained.

Score-assigning in our data differs from that in Kaplan and Zingales (1997) because we do not use information from the firm's balance sheets and income statements, neither do we use the firm's annual reports to shareholders (they are not available). The resulting likelihood of being credit constrained in this paper appears more accurately to reflect the underlying credit market frictions since we rely on observed outcome of financing. The Kaplan-Zingales index is more about the firm's need for external financing, and less about the degree to which the firm is constrained. The need for external financing depends on the demand for investment, as well as the firm's cash flow and cash stock. In the Kaplan-Zingales measure, businesses are unlikely to be financially constrained if their cash stock and cash flow (relative to the total assets) are high. In our measure, these businesses can be constrained if their requests for external financing are rejected or only partially approved.

3.2 Estimating likelihood of being credit constrained

With the measured degree to which firms are credit constrained, we now estimate to what extent firms' financial conditions and firm characteristics can be useful as proxy for credit constraints. This allows us to examine which proxy variables used in the literature are informative enough to infer whether credit constraints are binding. It is only with an observed measure of credit constraints that we are able to plausibly estimate the importance of proxy variables. In estimating the likelihood of binding credit constraints, we take into account that credit constraints are measured only for firms that needed external financing, and this need is endogenous. We therefore estimate the probability of being credit constrained in an ordered probit model with sample selection. The underlying latent variable is the cost of external financing relative to internal funds, which is unobserved but its impact (whether making firms constrained) can be inferred from the firm's observed variables.

Our first step is to compare financial conditions by the measured likelihood of being credit constrained, reported in Table 1. Relative median values between different degrees of financial constraints, unreported here, are qualitatively similar to relative mean values. Overall, financial conditions are relatively poor for firms that are likely constrained by credit. Constrained firms are small and young, have low cash flow and high current debt. Their sales growth is slow, and their demand for investment is low, relative to unconstrained firms. Investment-to-capital ratio among firms that reported not needing financing is close to the value for firms being mostly likely constrained, while the former have high cash flows and low debt, hinting that firms do not need financing because demand for investment is low.

Table 1: Median values by likelihood of being financially constrained, 2011

	Unlikely	Likely	Most likely	No value
Investment / Capital	0.13	0.09	0.02	0.04
CF / Asset	0.13	0.11	0.08	0.13
Current Debt / Asset	0.39	0.43	0.40	0.31
Long-term debt / Asset	0.11	0.07	0.09	0.04
Current Asset / Asset	0.61	0.66	0.70	0.70
Firm Age	12.00	10.00	9.00	14.00
Employment	7.00	5.00	4.00	4.00
Log (Capital)	11.60	10.90	10.40	10.50
Log (Asset)	13.10	12.70	12.00	12.04
Sales Growth	4.3%	3.3%	0.3%	0.6%
Dividend>0	0	0	0	0

Notes: The table reports the median values for variables used in this paper, except that the values for sales growth are mean due to data confidentiality. All values are obtained by applying the sample weights, thus the values are nationally representative. Variables are defined in Appendix C. In the last row, zero values indicate that the business does not have a positive dividend pay out.

3.2.1 Probit model with sample selection

Let f_{jt}^{*} be the cost of external financing relative to that of internal funds. This wedge is assumed to be a linear function of observables, including the ratio of current debt (short-term debt and current portion of long-term debt) to total assets $\frac{B^s_{jt}}{A_{jt}}$, the ratio of long-term debt to total assets $\frac{B^l_{jt}}{A_{jt}}$, the ratio of current (liquid) assets to total assets $\frac{CA_{jt}}{A_{jt}}$, the size of total assets $\ln(A_{jt})$, dividend $1_{\{\text{Div}_{jt}>0\}}$, the ratio of cash flow to total assets $\frac{CF_{jt}}{A_{jt}}$, and firm age Age_{jt}. These variables have often been frequently used in existing studies. All the stock variables are measured at the beginning of the year. We have

$$f^*_{jt} = a_1\frac{B^s_{jt}}{A_{jt}} + a_2\frac{B^l_{jt}}{A_{jt}} + a_3\frac{CA_{jt}}{A_{jt}} + a_4\frac{CF_{jt}}{A_{jt}} + a_5\ln A_{jt} + a_6 1_{\{\text{Div}_{jt}>0\}} + a_7\text{Age}_{jt} + a_8 g^o_{jt} + \varepsilon_{jt}. \tag{1}$$

We observe the outcome $f_{jt} = 0, 1,$ or 2, which reflects the underlying cost of external financing f^*_{jt}. A value of zero indicates that the firm is unlikely constrained, a value of one indicates that the firm is likely constrained, and a value of two indicates that the firm is most likely constrained. An ordered Probit model can be estimated to predict the probability of being constrained and the marginal effects.

Previous studies, for example, Kaplan and Zingales (1997) and Whited and Wu (2006), have also used other variables such as Tobin's q. There is a lack of information for calculating Tobin's q since most firms in our data sample are small and private. We instead use the sales growth g^o_{jt}.

The above outcome of being credit constrained f_{jt} is missing for more than half of the firms, as they did not select themselves to request external finance. This creates a sample selection issue, we observe the outcome only if firms needed or requested external financing. Let $D^f_{jt} = 0$ if the firm reported that it did not need external financing, and 1 otherwise. Thus, f_{jt} is observed if $D^f_{jt} = 1$. The decision as to whether the firm needs external financing is determined by the firm's demand for investment and its financial position, as well as the condition of credit markets. Variables affecting how binding credit constraints are should also affect the demand for external financing. Further, the firm's cash flow and sales growth should affect the demand for external financing through their effects on the demand for investment. Therefore, the demand for external financing is given by:

$$D^f_{jt} = \begin{cases} 1, & \text{if } \beta X > 0; \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Here,

$$\beta X = b_0 + b_1\frac{B^s_{jt}}{A_{jt}} + b_2\frac{B^l_{jt}}{A_{jt}} + b_3\frac{CA_{jt}}{A_{jt}} + b_4\frac{CF_{jt}}{A_{jt}} + b_5\ln A_{jt} + b_6 1_{\{\text{Div}_{jt}>0\}} + b_7\text{Age}_{jt} + b_8 g^o_{jt} + \mu_{jt}.$$

Equations (1) and (2) also include the interaction between a firm's age and its assets size. This allows to examine to what extent that young and small firms potentially are more likely to be constrained by credit. In our data set, a disproportionately large fraction of firms are small and old.

We estimate this model using the method described in Luca and Perotti (2011), which is implemented by the software Stata. As has become standard in sample selection models, the identification of parameters in the above model requires an exclusion restriction, i.e., Equation (2) contains at least variable that is not contained in Equation (1). To meet this restriction, we add to Equation (2) sector dummy variables, which are omitted in Equation (1). This implies that we are assuming that the demand for investment may differ by sector, say, due to the heterogeneity in productivity growth across sectors. Financial constraints may not depend on which sector the firm is in.[7]

It is noted that productivity is a main factor determining the demand for investment and employment, hence the need for financing. We do not use measures of productivity because productivity has to be estimated, which in turn should take into account financial constraints. Instead, cash flow, sales growth and other similar variables can reflect the underlying productivity and its growth.

3.2.2 Estimation results

Table 2 reports the main estimation results. Estimates from the ordered Probit model with sample selection are in the two columns under "Full model". In addition to the main estimation, we report additional two sets of supplementary estimation. Column (2) shows estimates using only the sample of firms that needed external financing, and column (3) shows estimates using the full sample but assuming that firms not needing external financing were unconstrained.

The estimation results confirm that some variables previously used in existing literature are indeed proper proxy for credit constraints among SMEs, while other variables are not strong indicator of credit constraints for SMEs. High cash flow is indeed a sign of being less likely constrained. Firms with a higher ratio of cash flow to assets are less likely to need financing, and less likely to be constrained if they do need it. This finding reconciles two arguments in earlier studies—one using the cash-flow sensitivity of investment as indirect evidence of credit market frictions, the other showing that such sensitivity can be an outcome in models without financial frictions. Our estimation suggests that both of these arguments only partially capture the role of cash flow. A high cash flow may indicate that the productivity level is high, the firm's investment is also high. These firms do not need external financing since high productivity and high cash flow suggest that the firm's financial condi-

[7]Our assumption that sectors differ in their demand for external financing is consistent with evidence from other countries by Rajan and Zingales (1998). These authors did not show whether the degree of financial constraints varies across industries.

tion is sound and able to support a high level of investment. On the other hand, a high cash flow may indicate or reveal the firm's project quality and overall firm performance, making the firm less likely to be constrained if it requests external financing.

Table 2: Ordered Probit estimation of financial constraint, 2011

	Full Model		(2)	(3)
	FC	NeedFin	FC	FC
Current debt / Assets	0.174***	0.264***	0.07	0.192***
	(0.057)	(0.038)	(0.049)	(0.039)
Long-term debt / Assets	0.003	0.153***	-0.067	0.063
	(0.040)	(0.053)	(0.058)	(0.046)
Current assets / Assets	0.007	-0.200**	0.084	-0.068
	(0.117)	(0.091)	(0.139)	(0.069)
ln(assets)	-0.201***	0.134**	-0.271***	-0.086
	(0.012)	(0.060)	(0.057)	(0.058)
Sales growth	-0.006	0.102	-0.04	0.044
	(0.047)	(0.075)	(0.072)	(0.033)
Cash flow / Assets	-0.211***	-0.128***	-0.193***	-0.201***
	(0.056)	(0.033)	(0.069)	(0.047)
$1_{\{Dividend>0\}}$	-0.192***	-0.01	-0.200***	-0.152**
	(0.069)	(0.048)	(0.072)	(0.067)
{3 < Age ≤ 5}	-0.629	0.715	-0.767	-0.1
	(1.262)	(1.354)	(1.455)	(1.485)
{5 < Age ≤ 10}	-1.284	-0.08	-1.172	-0.642
	(0.881)	(1.009)	(1.012)	(1.102)
{10 < Age ≤ 15}	-1.134**	0.237	-1.077	-0.529
	(0.533)	(0.956)	(1.101)	(0.685)
{15 < Age ≤ 20}	-1.107	0.131	-0.997	-0.705
	(0.690)	(1.006)	(0.766)	(1.046)
{20 < Age}	-2.554***	-0.359	-2.284**	-1.935**
	(0.461)	(0.863)	(1.130)	(0.765)
{3 < Age ≤ 5}*ln(assets)	0.06	-0.062	0.075	0.014
	(0.097)	(0.107)	(0.113)	(0.116)
{5 < Age ≤ 10}*ln(assets)	0.107	0.01	0.099	0.059
	(0.068)	(0.076)	(0.082)	(0.083)
{10 < Age ≤ 15}*ln(assets)	0.091**	-0.02	0.087	0.046
	(0.040)	(0.075)	(0.086)	(0.054)
{15 < Age ≤ 20}*ln(assets)	0.086	-0.021	0.081	0.053
	(0.054)	(0.081)	(0.062)	(0.082)
{20 < Age}*ln(assets)	0.188***	0.002	0.176**	0.136**
	(0.033)	(0.070)	(0.086)	(0.059)
Observations	7,258	7,258	3,747	7,397

Notes: Robust standard errors are in parentheses. For the p-values, * indicates the significance of coefficients at the 10% level, ** indicates 5% level, and *** 1% level. The table reports the main results of estimating measured financial constraints as a function of firm characteristics. Columns under "Full Model" report estimation by the ordered Probit with sample selection, where column "FC" reports the probability of three degrees of being financially constrained as specified by Equation (1), and column "NeedFin" reports the estimates of sample selection as in Equation (2). Column (2) reports the estimation of Equation (1) by the ordered Probit, using only the sample of firms reporting that they needed external financing. Column (3) reports the same estimation as in Column (2), but using the full sample and assuming that firms were financially unconstrained if they did not need external financing.

The debt-to-asset ratio is widely used as a proxy for credit constraints in both empirical studies and macroeconomics. The conventional view is that the higher the debt-to-asset ratio, the more likely the firm is constrained by external financing. Firms with a larger current debt-to-asset ratio are more likely to need external financing than otherwise, and they are more likely to be constrained if they do need financing. This points to the liquidity constraint. Firms with a larger long-term debt-to-asset ratio, though more likely to need external financing, are not significantly more constrained than otherwise.[8] In an alternative estimation using the total debt-to-asset ratio, not reported, the coefficient estimate for total indebtedness in the probability equation is statistically insignificant. On the assets side, the firm's ratio of current assets to total assets is statistically insignificant in indicating the degree of credit constraints, though firms with larger current assets tend to be less likely to need financing. Overall, coefficient estimates suggest that, for small and medium-sized firms, balance sheets are of limited use to proxy credit constraints. Information regarding financing activities and outcomes is more useful to directly measure the degree of credit constraints.

Firm size and age, often used in the literature, matter for indicating credit constraints. Estimation shows that larger and older firms are less likely to be credit constrained, conditional on that they need for external financing. Larger firms are also more likely to need external financing, while age does not appear to be important in determining the demand for external financing. This is consistent with models of collateral constraints, larger firms may have more collateral to use in obtaining external financing, this allows them to be more likely to request credit, and if they do, they are less likely constrained because they can provide more collateral. The marginal effects of assets size on firms being likely and most likely constrained are both negative and statistically significant, as shown in Table 3. Marginal effects are negative when the firm's age is 10 years or older, but statistically significant only if the firm is 20 years or older.

Table 3: Average marginal effects on probability of being credit constrained, 2011

	"Likely constrained"		"Most likely constrained"	
	dy/dx	Standard error	dy/dx	Standard error
ln(Asset)	-0.015	0.004	-0.017	0.006
Business Age<=2	0.022	0.024	0.017	0.033
Business Age>20	-0.029	0.019	-0.040	0.017
Current debt / asset	0.029	0.010	0.030	0.008
Long-term debt / asset	0.000	0.007	0.000	0.007
Cash flow / asset	-0.036	0.009	-0.036	0.011
Number of Observations, 7,258				

Notes: This table reports the average marginal effects based on the estimation by the ordered Probit with sample selection as reported in Table 2 under the column "Full Model". In calculating average marginal effects, predicted marginal probabilities are used, delta method is chosen for covariance matrix estimates, and sample weights are applied. Marginal effects for the probability of being "unlikely" constrained are not reported, although their signs are the opposite in Table 2.

[8] Using the 2007 data, we obtained a similar finding regarding the insignificance of long-term debt to asset ratio.

The full-model estimation separates the roles played by independent variables in indicating the firm's need for financing and the firm's likelihood of being constrained conditional on needing external financing. The correlation coefficient between the error terms of the two latent dependent variables, estimated at 0.63, is statistically significant. The likelihood-ratio test rejects the hypothesis that the errors for financial constraints and the errors for financial needs are uncorrelated. This correlation says that unobserved variables that increase the need for external financing also increase the probability of being credit constrained, justifying the ordered probit estimation with sample selection instead of the simple ordered Probit model, the latter is shown in columns (2) and (3) in Table 2.

About 57 percent of firms in 2011 did not need external financing, measure of credit constraints is missing for these firms. Since these firms did not need external financing, one could have assumed that they may be unconstrained. Estimation under this assumption is reported in column (3) of Table 2. This estimation is comparable to Kaplan and Zingales (1997) in the sense that it does not distinguish the need for external financing and the likelihood of being constrained conditional on having such a need. The probability of being constrained is again strongly and negatively correlated with cash flow, suggesting that cash flow is a significant indicator of being financially constrained regardless of the model specification. The key difference from the full model is that firm size alone is no longer statistically significant, because its two effects offset in this pooled estimation: the positive effect of firm size on the probability of needing external finance and the negative effect of firm size on the conditional probability of being financially constrained. This suggests that we cannot assume that firms not needing external finance are unconstrained, otherwise, firm size is unimportant in indicating financial constraints which appears rather surprising.

Average marginal effects reported in Table 3 are estimated using the average predicted marginal probabilities for each outcome, "likely constrained" and "most likely constrained". Results suggest that cash flow and current debt, both relative to asset, have strong impact on the probability of being constrained.

In summary, the estimation results suggest that variables such as cash flow, current debt, firm size, and dividend issuance are reasonable indicators as a proxy for credit constraints for small and medium-sized firms. Long-term debt or total debt is not informative to reflect credit constraints. The evidence also suggests that young firms are not obviously financially constrained, except that firms older than 20 years appear less constrained.

We should point out that the estimation relies on cross sectional variation in data for one year. This does not allow us to take into account time-varying factors at both firm and aggregate levels. One important factor is the supply of credit, which may vary from year to year. When credit supply is low, lenders may use credit rationing to reduce the supply of loans, making more firms constrained but due to lower supply. Another implication of using the cross-section data is that production function is estimated based on the likelihood of binding financial constraints from the out-of-sample prediction, which we discuss in more details in later sections.

4 Credit Constraints and Productivity

The importance of credit constraints lies in their impact on firm growth by restricting the firm's ability of investment and hiring. Firms' decisions also depend on productivity, which needs to be estimated. Total factor productivity is usually estimated through a structural estimation of firm-level production function, as in Olley and Pakes (1996) and extensions to their method.[9] In previous studies, the estimation of production function largely abstracts from credit constraints, while control variables (e.g., investment and employment) are functions of both total factor productivity and the firm's ability to finance its investment or employment. Therefore, omitting credit constraints leads to a bias of coefficient estimates for production function, which in turn leads to bias of productivity estimates. Below, we estimate production function by applying the method by Ackerberg, Caves, and Frazer (2015) to taking into account credit constraints. The estimated total factor productivity is then used as an independent variable to re-estimate the likelihood of being financially constrained. This allows us to examine the relationship between credit constraints and productivity.

4.1 Production function estimation with credit constraints

If credit constraints bind, investment and employment are likely to be lower than otherwise, conditional on the realization of a productivity shock. Abstracting from financial constraints in the model, low investment and employment are (mistakenly) considered as the outcome of a low productivity shock, leading to a downward bias in estimates of coefficients for labor and capital. The severity of this bias depends upon the firm's ability to finance investment and employment, as well as how many firms are constrained. Further, if productivity has an endogenous component, say partially determined by investment in research and development, the firm's inability to finance investment leads

[9] Such method is to overcome an upward bias in estimating production function by ordinary least squares (OLS), the bias is caused by the positive correlation between productivity and production factors (i.e., capital and labor). The main solution is to use observable endogenous variables such as investment, intermediate inputs, or employment as control for the unobserved productivity.

to a lower productivity. Therefore, to properly estimate productivity, credit constraints need to be taken into account.

Whether we can take into account credit constraints in estimating productivity depends on how credit constraints are specified. We focus on two of them—namely, the investment collateral constraint and the working capital loan collateral constraint. Let K_{jt} and B_{jt} be, respectively, firm j's capital stock and debt level at the beginning of period t. Let q_t be the asset price in period t, and X_{jt} be firm j's investment in period t, and let ω_{jt} be total factor productivity. The production function is $y_{jt} = F(\omega_{jt}, K_{jt}, L_{jt}, M_{jt})$, where L_{jt} and M_{jt} are, respectively, labor and intermediate inputs.

Investment collateral constraint. The constraint condition is given by

$$B_{jt+1} \leq \theta q_t K_{jt+1}, \tag{3}$$

under which investment is likely to be constrained. Parameter θ guides the degree of a binding constraint. The investment policy function in the firm's dynamic problem becomes

$$X_{jt} = \mathbf{X}(K_{jt}, B_{jt}, \omega_{jt}), \tag{4}$$

or a truncated value $X_{jt} = \overline{X}_{jt}$ if the collateral constraint binds. The optimal debt choice is $B_{jt+1} = \mathbf{B}(K_{jt}, B_{jt}, \omega_{jt})$, or $\overline{B}_{jt+1} = \theta q_t K_{jt+1}$ if the firm is constrained.[10]

If we follow Olley and Pakes (1996) to use investment as the control function, then $\omega_{jt} = \mathbf{X}^{-1}(X_{jt}, K_{jt}, B_{jt})$. This function has no closed form, and is approximated with high-order polynomials in the estimation procedure. Importantly, it is clearly that current debt B_{jt} is a state variable in a model with financing, even if the model abstracts from collateral constraints. This suggests that estimation that takes into account debt-asset ratio in previous studies does not catch the role of credit constraints. In the same vein, without relying on observed likelihood of credit constraints being binding, using investment as control function alone cannot overcome the estimation bias of omitting credit constraints.

We need to use the measured credit constraints in the control function, allowing us to isolate credit constraints from both financial conditions (e.g., debt-asset ratio) and unobserved productivity in shaping investment. By including measured credit constraints in control function, we can recover and resume the true correlation between productivity and inputs in the data, hence overcoming the bias of estimation. In addition, the selection bias due to endogenous firm exit is associated with fi-

[10] Here, we omitted other state variables such as age. See Appendix D for the full dynamic model.

nancing. More financially constrained firms are more likely to exit. The second stage of estimation in the Olley-Pakes method then needs to include measure of credit constraints in estimating the probability of firm exit.[11]

Collateral constraint of working capital. Firms may need external financing to pay wage bills. For example, a firm may have to pay workers before the labor input is employed for production or is irrespective of the firm's profitability, as in Neumeyer and Perri (2005) and Bianchi and Mendoza (2010). Suppose that the firm is allowed to borrow to pay a κ fraction of the labor cost, for which the firm pays interest. Total borrowing (investment loan and working capital loan) is subject to the collateral constraint as follows:

$$B_{jt+1} + \kappa w_t L_{jt} \leq \theta q_t K_{jt+1}.$$

In the firm's dynamic problem, the first-order optimal condition regarding labor input is then given by

$$\frac{\partial F(\omega_{jt}, K_{jt}, L_{jt}, M_{jt})}{\partial L_{jt}} = [1 + (R_t - 1 + \lambda^b_{jt})\kappa] w_t, \tag{5}$$

where $\lambda^b_{jt} \geq 0$ is the Lagrange multiplier for the collateral constraint. The optimal labor input is given by $L_{jt} = \mathbf{L}(K_{jt}, B_{jt}, \omega_{jt}; \lambda^b_{jt})$. Again, we need to have a measure of financial constraints in the data in order to separate the role of financial conditions as a state variable from that of credit constraints, because the value of λ_{jt} is endogenous and unobserved.

The estimation method by Ackerberg, Caves, and Frazer (2015) (hereafter ACF) can then be used, which we describe in the next section. Note that labor choice is static and does not depend on the lagged labor input L_{jt-1}. Here, we use the likelihood of being financially constrained measured and estimated in the data to proxy for λ^b_{jt}. This added information in estimation reduces the bias of productivity estimates.

4.2 Estimation procedure

Recognizing that the assumption of productivity-investment monotonicity likely breaks down in the data, in the presence of credit constraints, we use the intermediate input as a control function for unobserved productivity. This incorporates both forms of credit constraints discussed above. We use a two-stage procedure on the value-added production function, following ACF. In the first stage,

[11] Though, the short panel in our sample and the fact that all firms survived in the 2011 sample, make selection bias hard to overcome in estimation. Therefore, our estimation does not overcome the selection bias due to endogenous firm exit.

we substitute the inverted demand function of intermediate input into the production function, and estimate the part of output variation determined by inputs. In the second stage, all parameters of production function are estimated. This estimation strategy not only solves the collinearity problem in α_l in the first stage, raised by ACF, but also is appropriate in the case where both labor and capital inputs are dynamic choices.

Timing assumption. Entering period t, firm j's state variables are capital K_{jt}, debt B_{jt}, and employment L_{jt-1}. The firm first observes productivity ω_{jt}. The sequence of actions is then: the firm repays it current loan B_{jt} and interests, it then chooses future loan B_{jt+1} and makes decisions on employment L_{jt} and investment X_{jt}. Workers are paid before labor enters production. Finally, the firm chooses the optimal intermediate input M_{jt}, and production occurs.[12]

By the timing assumption, the intermediate input is a function of state variables and labor input (in lower cases after taking the natural logarithm, e.g., $m_{jt} = \ln M_{jt}$),

$$m_{jt} = M(k_{jt}, b_{jt}, l_{jt}, \omega_{jt}; \kappa, \theta).$$

Inverting this function and replacing parameters representing credit constraints with the measured likelihood of binding credit constraints, f^c_{jt}, we obtain ω_{jt} as a function of state variables and measured credit constraints,

$$\omega_{jt} = \mathbf{M}^{-1}(k_{jt}, b_{jt}, l_{jt}, m_{jt}, f^c_{jt}). \tag{6}$$

This allows for the possibility that a firm is financially constrained for both investment and employment. Note that f^c_{jt} is not interpreted as a production input, rather, it captures the firm-specific likelihood of being borrowing constrained, a factor affecting investment and employment that counter the positive correlation between these variables and unobserved productivity.

The value-added production function is given by

$$y_{jt} = \alpha_0 + \alpha_k k_{jt} + \alpha_l l_{jt} + \omega_{jt} + \varepsilon_{jt}. \tag{7}$$

Substituting for ω_{jt}, we obtain the equation of first-stage estimation as

$$y_{jt} = \alpha_0 + \alpha_k k_{jt} + \alpha_l l_{jt} + \mathbf{M}^{-1}(k_{jt}, b_{jt}, l_{jt}, m_{jt}, f^c_{jt}) + \varepsilon_{jt}, \tag{8}$$

[12] Note that L_{jt} is used for production in period t. Labor choice is assumed to be static, hence L_{jt-1} is not a state variable.

where a third-order polynomial is used for $\mathbf{M}^{-1}(\cdot)$.

It is important to emphasize our difference from existing studies, in the above production function. Financial variables and the measure of financial constraints are not taken as production inputs, nor as shifting factors of output, they affect the output level entirely through their impact on production inputs, capital and labor.

No parameters of production function are identified and estimated in the first stage, but we obtain the estimate $\widehat{\Phi}_{jt}$ of the composite term,

$$\Phi_{jt} = \alpha_0 + \alpha_k k_{jt} + \alpha_l l_{jt} + \mathbf{M}^{-1}(k_{jt}, b_{jt}, l_{jt}, m_{jt}, f^c_{jt}).$$

By controlling for endogenous production inputs, the first-stage estimation helps separate out the part of output determined by unanticipated shocks or the measurement error.

The productivity process is assumed to take the following form:[13]

$$\begin{aligned} \omega_{jt} &= g(\omega_{jt-1}, x_{jt-1}, f^c_{jt-1}) + \xi_{jt} \\ &= \sum_{i=0}^{3} \gamma_i (\omega_{jt-1})^i + \gamma_4 f^c_{jt-1} + \gamma_5 x_{jt-1} + \gamma_6 f^c_{jt-1} x_{jt-1} + \xi_{jt}. \end{aligned} \tag{9}$$

If $\gamma_4 = \gamma_5 = \gamma_6 = 0$, either by assumption or by estimation, then ω_{jt} is purely exogenous. The endogenous component of productivity is determined by investment and the degree of credit constraints in the lagged period. Lagged investment may affect productivity transition through innovation or added machines that embody new technology. Lagged likelihood of credit constraints can affect productivity through investment, captured by the interaction term.[14]

The second-stage estimation uses moment conditions of productivity shock ξ_{jt}. Given candidate values of $\alpha = (\alpha_k, \alpha_l)$, we obtain $\xi_{jt}(\alpha)$ as a residual from the regression of equation (9), in which

$$\omega_{jt}(\alpha) = \widehat{\Phi}_{jt} - \alpha_k k_{jt} - \alpha_l l_{jt}.$$

[13] In Equation 6, measure of credit constraints is an endogenous outcome which we "observe" in the data, it thus partly determined by both productivity and underlying frictions, just like investment. When inverted, the control function includes credit constraints. In Equation 9, it is assumed that the current-period productivity, which can affect whether a firm is financially constrained in current period, itself is affected by the lagged credit constraints. This process of endogenous productivity does not need to be used in the control function, because the underlying model is Markovian, current observed endogenous inputs are sufficient to serve as a control for productivity.

[14] Endogenizing productivity with lagged control variables has been used to study the contribution of R & D and exports on productivity, as in Aw, Roberts, and Xu (2011), De Loecker (2013), and Doraszelski and Jaumandreu (2013).

The moment condition is formulated as

$$E\left[\xi_{jt}(\alpha)\cdot \mathbf{Z}'_{jt}\right] = 0,$$

where $\mathbf{Z}_{jt} = \begin{pmatrix}1 & k_{jt} & l_{jt-1}\end{pmatrix}$ is a vector of instrument variables. Using this moment condition produces smaller variance and more stable estimates, also found by ACF, than using moment condition $E\left[(\xi_{jt}+\varepsilon_{jt})\cdot \mathbf{Z}'_{jt}\right] = 0$. We use the generalized method of moments (GMM) to estimate α. Once α is estimated, productivity ω_{jt} is calculated and its process can also be estimated. We choose capital stock and lagged labor input as instrument variables because these variables are already determined in period t, and they are correlated with the lagged productivity. For these variables to be valid instruments, reflecting serial correlation of productivity shocks, the capital returns and wages should be serial correlated, which we find they are in the data.

An alternative method of the second-stage estimation could be to estimate the following equation with the generalized method of moments:

$$\begin{aligned}\widehat{\phi}_{jt} &= \alpha_k k_{jt} + \alpha_l l_{jt} + \sum_{i=0}^{3} \gamma_i (\widehat{\phi}_{jt-1} - \alpha_k k_{jt-1} - \alpha_l l_{jt-1})^i \\ &\quad + \gamma_4 f^c_{jt-1} + \gamma_5 x_{jt-1} + \gamma_6 f^c_{jt-1} x_{jt-1} + \xi_{jt}. \end{aligned} \tag{10}$$

Note that, because $E_t[\xi_{jt} l_{jt}] \neq 0$, instrument variables are needed to overcome the endogeneity problem of labor choice. The instrument variables are the right-hand-side variables (except labor), lagged labor, lagged labor squared, and the lagged product of capital and labor.

Three issues of estimation need to be explained. First, the production function is estimated using the full data sample, but the likelihood of firms being credit constrained is observed and estimated for firms only in the year 2011. To overcome this limitation, we measure f^c_{jt} using the probability predicted by the above ordered Probit with sample selection, for all firms in the data set, both within sample (for the year 2011) and out of sample (for the rest of sample periods). Doing this way, the likelihood of binding financial constraints vary over time only to the extent independent variables vary in Equation (1).

Second, selection bias due to the omission of the exit decision is not considered. Although the data sample includes small and medium-sized firms, whose probability of exit is greater than large firms, the sample is short, spanning from 2009 to 2013, and all firms in the sample survived in 2011 (the survey year). The firm exit rate is fairly small. Incorporating firm exit in estimation is left for future work.

Third, our estimation pools firms in all sectors. Goods-producing firms account for a small share of the total number of firms, as most firms are in retail trade and services. Estimation by sector may create a small-sample problem, since only 22 percent of firms are constrained in 2011. To take into account differences in productivity across sectors, we augment the first-stage estimation (equation (6)) to include the real hourly wage rate at the level of three-digit NAICS and the price of intermediate inputs (relative to aggregate GDP price) at the level of two-digit NAICS.[15]

4.3 Diagnostics

We first examine whether credit constraints and productivity are correlated, based on the productivity estimated without taking into account financing. In estimating the second-stage equation, we use capital, lagged labor, and the square of lagged labor as instrument variables.

Parameter estimates of the production function are reported in Table 4. The first row shows the estimation by the ordinary least squares, the second row is the estimation assuming that $\gamma_4 = 0$, and the third row is the estimation with endogenous total factor productivity ($\gamma_4 \neq 0$). The estimate of labor coefficient in the two-stage estimation without financing is smaller than in ordinary least squares (OLS), because OLS estimation does not take into account the positive correlation between productivity and labor choice. This positive correlation causes an upward bias of coefficient estimate of labor input. The coefficient estimate for capital is larger in two-stage estimation than in OLS, also as expected. By inverting the optimal intermediate input to obtain the proxy for total factor productivity, conditional on the intermediate input, total factor productivity is smaller for firms with larger capital size. Thus, the coefficient estimate for capital is expected to be larger when moving from OLS to two-stage estimation.

To diagnose the relationship between productivity and credit constraints, we use the estimated productivity without taking into account financing and assume that the productivity process is exogenous. Estimated productivity and measured credit constraints are negatively correlated, with a correlation coefficient of -0.3. An OLS regression of the estimated productivity on the measured probability of being constrained and its lag also shows that productivity is correlated negatively with measured credit constraints. When using assigned scores for the 2011 data sample, the median of estimated productivity among firms likely constrained and most likely constrained is, respectively, 11 percent and 19 percent lower than that of firms unlikely constrained.

[15] We use firms from sectors of NAICS 23, 31-33, 41, 44-45, 48-49, 54, 55, 56, 72 and 81. Sector-level real wage rates and relative prices of intermediate inputs are calculated using the tables from Survey of Employment, Payrolls and Hours (SEPH) and the Industrial Producer Prices Index (IPPI), these tables are publicly available.

Table 4: Production function estimation with different specifications

	$\widehat{\alpha_l}$	Std.Err.	$\widehat{\alpha_k}$	Std.Err.	RHS vars. in TFP shock process
(i) No financing decisions					
OLS	0.715	0.008	0.049	0.004	-
(ii) No financing decisions					
2-stage, exog	0.652	0.166	0.054	0.013	-
2-stage, endo	0.631	0.091	0.052	0.009	X_{jt-1}
(iii) With external financing, but without financial constraint					
2-stage, exog	0.665	0.195	0.057	0.011	-
2-stage, endo	0.639	0.206	0.056	0.011	$\widehat{f^c}_{jt-1}$
2-stage, endo	0.623	0.203	0.056	0.011	$\widehat{f^c}_{jt-1}, X_{jt-1}, \widehat{f^c}_{jt-1} * X_{jt-1}$
(iv) With financial constraint					
2-stage, exog	0.664	0.101	0.059	0.011	-
2-stage, endo	0.683	0.370	0.061	0.020	$\widehat{f^c}_{jt-1}$
2-stage, endo	0.674	0.280	0.061	0.013	$\widehat{f^c}_{jt-1}, X_{jt-1}$
2-stage, endo	0.672	0.268	0.061	0.012	$\widehat{f^c}_{jt-1}, X_{jt-1}, \widehat{f^c}_{jt-1} \cdot X_{jt-1}$
2-stage, endo	0.643	0.244	0.057	0.013	$\widehat{f^c}_{jt-1}, X_{jt-1}, \widehat{f^c}_{jt-1} \cdot X_{jt-1}, \widehat{f^c}_{jt-1} \cdot \omega_{jt-1}$

Notes: This table reports estimation of production function as in Equation (7) by various methods, from the OLS to GMM estimation with financial constraints. Estimation (i) uses the OLS on Equation (7), ignoring the simultaneity bias and financing. Estimations (ii) to (iv) use the two-stage estimation with GMM, where (ii) abstracts from external financing, (iii) allows for external financing, but abstracts from financial constraints, and (iv) imposes financial constraints. In each specification, we allow that the productivity process is exogenous indicated by "exog", and endogenous indicated by "endo". The specification of productivity processes is shown in the column far right.

The negative correlation between estimated productivity and measured credit constraints suggests two possibilities. First, productivity estimates are biased in this case because credit constraints are omitted, given that both investment and employment are negatively correlated with measured credit constraints. If credit constraints are omitted from the estimation, low investment or low employment in the data is an outcome of low productivity, while in fact this could be due to binding credit constraints. Thus, if estimation does not correct for the negative correlation between inputs and credit constraints, estimated productivity is biased upward, particularly for firms that are constrained, leading to a negative correlation between productivity and the measured credit constraints.

Second, the negative correlation between estimated productivity and measured credit constraints also suggests that productivity, through its effect on investment and employment, may indirectly affect the probability of being financially constrained. This is possible if firms misreported their activities related to external financing. The rejection of a loan request could be a result of low productivity instead of financial frictions. To see if this is the case, we add estimated total factor productivity as an independent variable and re-estimate the probability of being financially constrained.

The results are reported in Table 5.[16] Clearly, more productive firms, conditional on needing external financing, are less likely to be financially constrained. Further, compared with the estimation in Table 2, the magnitude of the coefficient estimates for firm assets size, the cash-flow-to-asset ratio, and dividends becomes smaller when total factor productivity is taken into account. This suggests that variables such as cash flow, dividends, and firm size reflect partially the level of firm productivity. If we use them to proxy for credit constraint, we may not be able to distinguish low productivity from binding credit constraints. However, the coefficient estimates for total factor productivity in Table 5 may no longer be statistically significant once productivity is properly estimated by incorporating the firm's financing decision and credit constraints—which is the case, as we will show later.

Table 5: Ordered probit estimation of financial constraint, 2011: role of TFP

	(1)		(2)	
	FC	NeedFin	FC	NeedFin
Current debt / Assets	0.195***	0.274***	0.181**	0.289***
	(0.066)	(0.053)	(0.074)	(0.063)
Long-term debt / Assets	0.001	0.148***	0.055	0.158**
	(0.056)	(0.051)	(0.069)	(0.071)
Current assets / Assets	0.0646	-0.197**	0.008	-0.236**
	(0.108)	(0.083)	(0.101)	(0.092)
ln(assets)	-0.086***	0.209***	-0.078***	0.205***
	(0.011)	(0.074)	(0.012)	(0.075)
Sales growth	-0.038	0.238***	-0.042	0.202**
	(0.107)	(0.055)	(0.092)	(0.084)
Cash flow / Assets	-0.146***	-0.102*	-0.187***	-0.080
	(0.055)	(0.061)	(0.036)	(0.076)
$1_{\{Dividend>0\}}$	-0.177***	-0.029	-0.114**	-0.010
	(0.068)	(0.044)	(0.056)	(0.040)
TFP	-0.269***	-0.080		
	(0.091)	(0.097)		
Lagged TFP			-0.308***	-0.120
			(0.093)	(0.090)
# of Obs.	6,459	6,459	6,112	6,112

Notes: Robust standard errors are in parentheses. For the p-values, * indicates the significance of coefficients at the 10% level, ** indicates 5% level, and *** 1% level. The table reports the estimated effect of total factor productivity (TFP) on measured financial constraints. Columns (1) and (2) have the same specification but one uses contemporaneous TFP and the other uses lagged TFP. The estimation is by the ordered Probit with sample selection, where column "FC" reports the probability of three degrees of being financially constrained as specified by Equation (1), and column "NeedFin" reports the estimates of sample selection as in Equation (2). Coefficient estimates for terms of firm age, and interaction of firm age and size are not reported. Total factor productivity (TFP) was estimated in the model abstract from external financing, as shown in case (ii) "exog" in Table (4).

[16] Here and in Table 6, total factor productivity is a generated regressor. Standard errors and test statistics are in general asymptotically invalid, hence should be adjusted. See Pagan (1984) and Wooldridge (2010) for discussions. We do not adjust standard errors, considering that we do not conduct formal statistical tests.

Table 6: Ordered probit of financial constraint, 2011: role of TFP

	(1)		(2)	
	FC	NeedFin	FC	NeedFin
Current debt / Assets	0.190***	0.263***	0.194***	0.274***
	(0.074)	(0.051)	(0.065)	(0.051)
Long-term debt / Assets	0.026	0.158***	0.015	0.174***
	(0.047)	(0.054)	(0.055)	(0.051)
Current assets / Assets	0.045	-0.184**	0.038	-0.188**
	(0.113)	(0.086)	(0.115)	(0.080)
ln(assets)	-0.139***	0.180**	-0.106***	0.242***
	(0.013)	(0.070)	(0.011)	(0.075)
Sales growth	0.018	0.214***	-0.053	0.203***
	(0.091)	(0.053)	(0.099)	(0.062)
Cash flow / Assets	-0.184***	-0.083*	-0.159***	-0.089
	(0.053)	(0.046)	(0.052)	(0.060)
$1_{\{\text{Dividend}>0\}}$	-0.190***	-0.018	-0.172**	-0.020
	(0.066)	(0.037)	(0.069)	(0.042)
TFP	-0.033	-0.145**		
	(0.087)	(0.072)		
Lagged TFP			-0.072	-0.186***
			(0.090)	(0.063)
# of Obs.	6,459	6,459	6,112	6,112

Notes: Robust standard errors are in parentheses. For the p-values, * indicates the significance of coefficients at the 10% level, ** indicates 5% level, and *** 1% level. The table reports the estimated effect of total factor productivity (TFP) on measured financial constraints. Coefficient estimates for terms of interaction of firm age and size are not reported. The only difference from Table (5) is that here total factor productivity (TFP) was estimated in the model incorporating financial constraints, as shown in case (iv) "exog" in Table (4).

4.4 Results of production function estimation

The last two sets of estimates in Table 4 correspond to the estimation of production function, taking into account external financing (debt as a state variable) and measured credit constraints. In the case of exogenous productivity, coefficient estimates of both labor and capital are larger than estimates when external financing and financial constraints are omitted. This is expected, since the correlation is negative between credit constraints and production factors. When the two-stage GMM estimation is augmented to incorporate credit constraints, the true correlation between total factor productivity and inputs are recovered, which in turn makes the coefficient estimate of labor larger. For example, a productive firm observes a low labor input (relative to other firms with the same level of total factor productivity) owing to a binding borrowing constraint. If this constraint is omitted from the estimation, total factor productivity is then the only factor determining labor input, the correlation between productivity and labor input will appear weak. Once the measure of credit constraints is taken into account, true correlation between productivity and labor input is then resumed, leading to a coefficient estimate for labor that is larger than without financing. A similar argument applies to the coefficient estimate for capital.

Comparing the model with external financing but without credit constraints and the model with credit constraints, again for the case of exogenous productivity, coefficient estimates for labor and capital are slightly larger in the latter model. This is likely caused partly by the way we measure the likelihood of credit constraints. In the two-stage GMM estimation, proxy variables for total factor productivity, such as firm size and current debt, were also used to estimate the likelihood of credit constraints. Given the significance of firm size and indebtedness in predicting financial constraints, it may not be surprising that coefficient estimates are only slightly larger after measured credit constraints are incorporated. Further, for sample years other than 2011, we use the probability of being financially constrained that is predicted out of sample by the ordered Probit model, these predicted probabilities capture credit constraints primarily to the degree explained by firm size and indebtedness. Ideally, this issue can be resolved if we were able to assign scores of financial constraints for firms in each sample period, so that we do not need to use the firm's size and indebtedness to estimate and predict the likelihood of being constrained.

Estimated total factor productivity (de-meaned, $\omega_{jt} - \overline{\omega}$) and measured credit constraints are still negatively correlated, with a correlation coefficient of -0.45, stronger than the -0.30 obtained in the case of estimation abstracting from external financing. In 2011, the median total factor productivity of the likely constrained firms is 9 percent lower than that of the unconstrained firms, and the median total factor productivity of the most likely constrained firms is 19 percent lower than that of unconstrained firms. These differentials are close to those of the productivity estimates when external financing is ignored. The negative correlation between the estimated productivity and the measured credit constraints suggests that our measure of credit constraints labels firms being financially distressed as financially constrained, hence cannot distinguish the two cases: distressed due to being highly indebted and being less productive; constrained due to credit market frictions.

However, the negative correlation between measured credit constraints and estimated total factor productivity is now reflected more in the impact of total factor productivity on the demand for external financing, and less in the impact on the conditional probability of being financially constrained (see Table 6). This is in contrast to estimates in Table 5, where the negative correlation is seen mostly through the impact of total factor productivity on the conditional probability of being constrained.

This result appears to be intuitive. Once the bias of the production function estimation is corrected by taking into account external financing and credit constraints, information on financial frictions (or part of it) contained in productivity estimates is removed. Therefore, a low level of estimated productivity is no longer indicative of a high probability of being constrained.

4.5 Bias of estimated productivity

We now compare estimates of total factor productivity under different specifications. Since moments of the error term may differ across estimates, to make them comparable, total factor productivity here is calculated as $\exp(y_{it} - \widehat{\beta}_k k_{it} - \widehat{\beta}_l l_{it})$.

Table 7 reports mean values of estimated total factor productivity. Overall, when external financing and credit constraints are omitted from the production function estimation, total factor productivity is biased upward by about 4 percent. The magnitude of this bias is similar among firms of different sizes and those with different probabilities of being credit constrained. The bias of median values is similar to that of mean values. Table 7 also shows that the gap in average productivity between constrained and unconstrained firms did not change after the bias was corrected. This implies that the productivity distribution is not significantly changed after the bias is corrected.

Table 7: Comparison of productivity estimates, 2011

	OLS	2-stage		
		No financing	With financing, no FC	With FC
Overall	5.86	6.37	6.25	6.14
		By assets size quintile		
1 (small)	5.56	6.02	5.91	5.81
2	5.68	6.15	6.04	5.94
3	5.83	6.31	6.21	6.10
4	5.96	6.51	6.37	6.25
5 (large)	6.28	6.85	6.73	6.60
		By measured FC		
Unlikely	5.96	6.49	6.36	6.24
Likely	5.90	6.41	6.29	6.18
Most likely	5.72	6.21	6.10	5.99
No value	5.84	6.34	6.22	6.11

Notes: This table reports the weighted mean values of firm-level total factor productivity (TFP), the latter is calculated as $\exp(y_{it} - \widehat{\beta}_k k_{it} - \widehat{\beta}_l l_{it})$. Coefficient estimates $\widehat{\beta}_k$ and $\widehat{\beta}_l$ are obtained with estimation reported in Table (4), for cases of TFP being exogenous.

The estimated bias here could be a lower bound, because our measure of credit constraints could potentially have underestimated the fraction of firms being credit constrained. Recall that we labeled a firm as being unconstrained if its request for external financing is fully approved, such a fully-approved request could have been made by the business owner whose decisions take into account the underlying financial frictions.

4.6 Financial constraint and endogenous productivity

So far, the discussion has been focusing on exogenous total factor productivity. Binding credit constraints can cause a lower productivity by limiting the firm's activities intended to boost productivity. We estimate whether the lagged probability of being credit constrained can affect total factor productivity growth. Specifically, in estimating the production function, the productivity process is endogenous to lagged investment and the lagged probability of being credit constrained. After obtaining productivity estimates, we use ordinary least squares to estimate the productivity processes.

To diagnose whether measured credit constraints affect productivity, we estimate the production function with external financing but assume no credit constraints, so that in the first stage of the GMM estimation, we use $\omega_{jt} = \mathbf{M}^{-1}(k_{jt}, b_{jt}, l_{jt}, m_{jt})$ to proxy for productivity. In the second stage, productivity is endogenous. Estimates reported in Table 8 show that a higher likelihood of being credit constrained leads to a lower productivity.

Table 8: Estimation of endogenous productivity

	Coefficient	Robust Std.Err.
ω_{jt-1}	-1.326	1.174
ω^2_{jt-1}	0.374	0.167
ω^3_{jt-1}	-0.020	0.008
$\widehat{f^c}_{jt-1}$	-0.139	0.036
x_{jt-1}	0.0001	0.0004
$x_{jt-1} \cdot \widehat{f^c}_{jt-1}$	0.001	0.002
Constant	4.996	2.743
# of obs.	25,966	
F test:	$F(6,15) = 1305.5$	
R-square:	0.802	

Note: This table reports an OLS estimation of the endogenous process of TFP. TFP was estimated by the 2-stage GMM procedure abstracting from financial constraints but allowing for external financing. In the 2-stage GMM estimation, the TFP process is specified as in this table.

However, when the production function is estimated by incorporating external financing and credit constraints, that is, $\omega_{jt} = \mathbf{M}^{-1}(k_{jt}, b_{jt}, l_{jt}, m_{jt}, f^c_{jt})$ is used in the first stage, various estimates of γ_4 are all positive and statistically insignificant (as shown in Table 9). We take this as evidence that, once the estimation bias from omitting financial constraints is corrected, the measured likelihood of being constrained no longer has a separate effect on productivity processes. In an extended estimation, we interact measured credit constraints with lagged productivity. Table 10 shows that firms with a higher lagged productivity and a larger probability of being constrained are less productive, resulting in a lower productivity growth.

In summary, we find no strong evidence of the effect of credit constraints on productivity, after the productivity estimation corrects the bias due to the omission of credit constraints.

Table 9: Estimation of endogenous productivity

	Coefficient	Robust Std.Err.
ω_{jt-1}	-2.515	1.494
ω^2_{jt-1}	0.567	0.239
ω^3_{jt-1}	-0.031	0.013
$\widehat{f^c_{jt-1}}$	0.065	0.040
x_{jt-1}	-0.0001	0.0004
$x_{jt-1} \cdot \widehat{f^c_{jt-1}}$	0.001	0.002
Constant	7.388	3.159
# of obs.	19,099	
F test:	$F(6,15) = 1166.6$	
R-square:	0.746	

Notes: Note: This table reports an OLS estimation of the endogenous process of TFP. TFP was estimated by the 2-stage GMM procedure taking into account financial constraints. In the 2-stage GMM estimation, the TFP process is specified as in this table.

5 Productivity, credit constraint and firm growth

If credit constraints are believed to lead to misallocation, hence productivity loss, we should expect that credit constraints affect the firm growth, in terms of both capital stock and employment. Firms that are more likely constrained by the external financing are expected to experience a slower growth, resulting in misallocation and productivity loss.

We first focus on the real effect of the estimated degree to which firms are credit constrained. Decisions on investment and employment are determined by the underlying productivity shocks, factor adjustment costs, and the likelihood of being credit constrained. The importance of measured

Table 10: Estimation of endogenous productivity

	Coefficient	Robust Std.Err.
ω_{jt-1}	-2.058	1.510
ω^2_{jt-1}	0.511	0.234
ω^3_{jt-1}	-0.028	0.012
x_{jt-1}	-0.0002	0.0004
$\widehat{f^c}_{jt-1}$	2.122	0.640
$x_{jt-1} \cdot \widehat{f^c}_{jt-1}$	0.002	0.002
$\omega_{jt-1} \cdot \widehat{f^c}_{jt-1}$	-0.331	0.105
Constant	5.996	3.311
# of obs.	19,099	
F test:	$F(7,15) = 1890$	

Notes: Note: This table reports an OLS estimation of the endogenous process of TFP. TFP was estimated by the 2-stage GMM procedure taking into account financial constraints. In the 2-stage GMM estimation, the TFP process is specified as in this table, which is different from that in Table (9).

credit constraints in our model should be able to be separated from other variables, such as cash flow that is indicative of both the level of productivity and the likelihood of being credit constrained.

We estimate the reduced-form equations for investment and employment growth, implied by the firm's Euler equation. The data sample spans from 2008 to 2013, while the measure of credit constraints is only available for 2011. We therefore use the estimated model of credit constraints to predict the probability of a firm being credit constrained for the years before and after 2011. These predicted probabilities are unconditional on the demand for external financing, because this demand is unobserved other than in 2011.

The estimation of investment-to-capital ratio in a linear equation, for which results are not reported here, shows that parameter estimates are statistically insignificant. One possible reason for this the substantial heterogeneity of investment by small and medium-sized firms. Using definitions of inaction and lumpiness of investment in Cooper and Haltiwanger (2006), we find that 31 percent of the firm/year sample displays an investment-to-capital ratio of within 1 percent in absolute value. The fraction of zero investment is 46 percent among firms in the first lowest quintile of assets size, in contrast to only 17 percent in the last largest quintile. On the other end, about 33 percent of the sample displays an investment-to-capital ratio larger than 20 percent. The linear regression becomes insignificant, given the substantial inaction and spike. In addition, measurement errors of investment and capital stock are a potentially more severe problem among these small firms used in our sample, compared with large firms used in previous studies.

We therefore estimate the extensive margin, the probability of a positive investment. Table 11 reports coefficient estimates of a logit model under five specifications.[17] The data sample spans from 2008 to 2013, and contains firms from most business sectors of the two-digit NAICS. Age and sector dummy variables are included in all specifications (not reported). The fully specified estimation is reported in column (5). Investment is sensitive to both cash flow and measured credit constraints. More likely constrained firms with a low cash flow are less likely to invest a positive amount. In addition, the likelihood of a positive investment increases if the current debt-to-asset ratio becomes greater. Estimates in column (1) represent a model specification similar to investment regressions in earlier studies. Investment, as usual, is sensitive to cash flow; firms with a higher cash flow are more likely to invest. The total debt-to-asset ratio is insignificant and its coefficient has an opposite sign if this ratio is believed to reflect the degree of financial constraint. In column (2), where only the measured unconditional probability of being credit constrained is included, we observe that the more likely the firm is constrained, the less likely its investment is positive.

Employment growth is also sensitive to measured credit constraints. Table 12 reports results of linear regressions of employment growth rate on financial variables and measured credit constraints. Coefficient estimates are again consistent with our prior—credit constraints limit firm growth. In addition, employment grows faster for firms with a higher cash flow. These findings are by and large comparable to the regression of extensive margin of investment.

Adding the estimated productivity to the firm growth regression does not change the above results. Focus on the employment growth, Table 13 reports estimation from a linear regression, where the dependent variable is the percentage change in employment. Clearly, when taking into account TFP estimates, the degree to which firms are financially constrained continues to have a negative impact on employment growth. This and the above evidence are in line with previous literature, for example, Angelini and Generale (2008) and Cabral and Mata (2003). Coefficient estimates for total factor productivity are negative but statistically insignificant for lagged total factor productivity, similar to results based on sector-level data by Basu, Kimball, and Fernald (2006).[18] Finally, the extensive margin of investment is adversely affected by the measured degree of financial constraints, and positively affected by total factor productivity, both estimates are statistically significant.

[17] Again, in the reported estimation here and in other tables of this section, measure of credit constraints and total factor productivity are generated regressors. We did not adjust standard errors of coefficient estimates.

[18] Not reported here, coefficient estimates for the change in total factor productivity are negative and statistically significant. A potential explanation for the negative effect of total factor productivity on employment growth is nominal rigidity in output prices.

Table 11: Investment regression, extensive margin, 2009 to 2013

	(1)	(2)	(3)	(4)	(5)
X_{jt-1}/K_{jt-1}	-0.0005*	-0.0004	-0.0005*	-0.0005**	-0.0005**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
CF_{jt}/A_{jt}	0.199**		0.195**	0.059	0.020
	(0.083)		(0.097)	(0.070)	(0.036)
B_{jt}/A_{jt}	0.006		0.007	0.009	
	(0.024)		(0.023)	(0.023)	
ln(assets)	0.173***		0.170***	0.174***	0.163***
	(0.018)		(0.025)	(0.023)	(0.022)
Prob(FC)		-1.429***	-0.068	-0.077	-0.537**
		(0.178)	(0.308)	(0.245)	(0.240)
Prob(FC)*CF_{jt}/A_{jt}				0.737***	0.728***
				(0.162)	(0.139)
B^s_{jt}/A_{jt}					0.071***
					(0.025)
Constant	-1.864***	0.788***	-1.811***	-1.866***	-1.607***
	(0.246)	(0.062)	(0.421)	(0.370)	(0.348)
Observations	33,634	33,951	33,634	33,634	33,611

Notes: Robust standard errors are in parentheses. For the p-values, * indicates the significance of coefficients at the 10% level, ** indicates 5% level, and *** 1% level. The table reports the coefficient estimates in the logit model, the dependent variable is probability investment-to-capital ratio being positive. TFP is estimated in the model with financial constraint, assuming that the TFP process is exogenous.

6 Conclusions

In this paper, we constructed a measure of the likelihood of firms being constrained by external financing, using detailed information on activities and outcomes of financing among small and medium-sized firms in Canada. This measure appears to reflect financial frictions facing borrowing firms. The advantage of our measure is that it separates the need for external financing from the likelihood of being constrained by borrowing, which is missing from previous studies. We found that a firm's assets size and cash flow are robust indicators of credit constraints, while the total debt-to-asset ratio is not. This result raises a caveat for models using the debt-to-asset ratio to be the proxy for credit constraints.

We also show that coefficient estimates for capital and labor in production function are downward biased if debt and measured credit constraints are omitted, because of negative correlations between measured credit constraints and production inputs. This downward bias in turn leads to an upward bias in the productivity estimation. Without correcting the bias, estimated productivity is

Table 12: Employment growth and financial constraint, 2009 to 2013

	(1)	(2)	(3)	(4)	(5)
CF_{jt}/A_{jt}	0.007***		0.006***	0.006***	0.006***
	(0.001)		(0.001)	(0.001)	(0.001)
B_{jt}/A_{jt}	-0.023***		-0.022***	-0.023***	
	(0.005)		(0.005)	(0.005)	
ln(assets)	-0.005*		-0.010***	-0.006	-0.0002
	(0.003)		(0.003)	(0.004)	(0.003)
Prob(FC)		-2.185***	-0.153***	-0.062	0.062
		(0.142)	(0.043)	(0.064)	(0.055)
Prob(FC)*CF_{jt}/A_{jt}				0.119**	0.154***
				(0.048)	(0.038)
B^s_{jt}/A_{jt}					-0.034***
					(0.006)
Constant	0.183***	1.056***	0.296***	0.202**	0.092
	(0.049)	(0.025)	(0.055)	(0.081)	(0.062)
Observations	41,917	58,540	41,917	41,917	41,896
R-squared	0.014	0.204	0.014	0.015	0.015

Notes: Robust standard errors are in parentheses. For the p-values, * indicates the significance of coefficients at the 10% level, ** indicates 5% level, and *** 1% level. The table reports the coefficient estimates in a linear regression of employment growth. TFP is estimated in the model with financial constraint, assuming that the TFP process is exogenous.

negatively correlated with measured credit constraints. This is reflected in the statistically significant coefficient estimates pointing to a negative effect of productivity on the conditional probability of being constrained. Once productivity is properly estimated, such a correlation, although still negative, no longer suggests that productivity affects the conditional probability of being constrained. Rather, it is reflected in the impact of productivity on the need for financing.

Results from this paper have some policy implications. Our findings suggest that government may want to condition its policy such as subsidies for small businesses on their performance in productivity and productivity growth. Small firms with a great potential in terms of productivity, if constrained by external financing, are unable to grow, contributing to a productivity loss at the aggregate level. Government assistance in overcoming financial difficulties to these firms can then help boost aggregate productivity, such policy can have a prolonged impact on economic growth. A policy that is conditional only on firm size is less effective, because some small firms that are financially constrained are also less productive, assisting them in overcoming financial distress does not necessarily improve their performance in terms of growth and survival probability. Further, policies towards overcoming liquidity shortage appear effective to help small businesses walk out of financial stress.

Table 13: Impact of FC and TFP on employment growth

	(1)	(2)	3)	(4)
CF_{jt}/A_{jt}	0.009	0.007		
	(0.009)	(0.018)		
B^s_{jt}/A_{jt}	-0.016***	-0.010**		
	(0.004)	(0.004)		
ln(assets)	-0.0004	-0.003		
	(0.004)	(0.003)		
Prob(FC)	-0.072	-0.067	-0.155**	-0.100*
	(0.065)	(0.060)	(0.064)	(0.050)
TFP	-0.024		-0.028**	
	(0.016)		(0.012)	
Prob(FC)*CF_{jt}/A_{jt}	0.116**	0.094*		
	(0.046)	(0.053)		
Lagged TFP		0.021		0.014
		(0.012)		(0.009)
Constant	0.055	0.062	0.076***	0.040**
	(0.060)	(0.039)	(0.024)	(0.015)
Observations	32,992	25,827	33,227	26,011
R-squared	0.009	0.007	0.005	0.004

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: Robust standard errors are in parentheses. For the p-values, * indicates the significance of coefficients at the 10% level, ** indicates 5% level, and *** 1% level. The table reports the coefficient estimates in a linear regression of employment growth. Firm age and industry are controlled for, but not reported. TFP is estimated in the model with financial constraint, assuming that the TFP process is exogenous.

To what extent a firm is credit constrained is ultimately an endogenous outcome, given the demand for financing. Demand for external financing is determined by the need for investment and employment which in turn is determined by the underlying productivity shocks. To estimate both the likelihood of being financially constrained and productivity processes in one framework requires estimating a fully-specified dynamic model of investment and financing, incorporating a theory of financial constraint.

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Appendix A Data Description

Overall, small and medium-sized enterprises (SMEs) (firms with fewer than 500 employees) account for more than 50 percent of business sector output, and 70 percent of hours worked (2008 data (Rispoli, Leung, and Baldwin, 2013)).

We use data from the Survey on Financing and Growth of Small and Medium Enterprises (SFSME) merged with administrative data. The SFSME is a repeated cross-section database of Canadian firms with fewer than 500 employees and with gross revenue below $50 million. It provides detailed information on a firm's demand for financing, reported demand for external financing, reasons the loan request is rejected, how the firm uses the loan, the borrowing rate and rate type, fees for obtaining credit, collateral and whether the loan is guaranteed by the government. The data also provide information on leasing, equity financing, and government financing. Data are available for the years 2004, 2007, and 2011. The administrative data provide information on firms in the SFSME regarding financial statements and income statements.

In this paper, we use data from the 2011 survey, although we also report financing activities from the 2007 survey.

For our purpose, we keep firms with two-digit NAICS 23, 31-33, 41, 44-45, 48-49, 54, 55, 56, 72 and 81. The excluded firms are from sectors of agriculture, finance and insurance, real estate, rental and leases, education, utilities, and health services. To make the final data set for estimation, we set negative values to be missing for variables such as sales, assets, and liabilities. We also set missing values respectively for the top and the bottom 0.5% of observations for variables of ratios, such as investment-capital ratio, cash flow to asset ratio, debt to asset ratio. The raw firm count is close to 10,000. Each firm is assigned with a weight, making the weighted sample representative of the firm population. In all calculations and estimation, we apply the weights. The final data set has 9,109 observations for the year 2011.

A.1 Main survey questions

Below is the list if main variables on financing activities and outcomes, used for measuring financial constraint.

1. In 2011, which types external financing did your business seek?

2. Why did your business not seek external financing in 2011?

3. For your business' largest request for debt financing in 2011, what was the dollar amount requested?

4. What was the outcome of this debt financing request?

5. What reasons were given by the credit provider for turning down the request for debt financing?

6. What collateral was your business asked to provide to obtain this new debt financing in 2011?

Questions of 3, 4, and 5 were also asked for other types of external financing, including lease financing, trade credit financing, equity financing, and government financing.

A.2 Summary statistics on financing, 2011

There are five financing instruments: loans, equity, leasing, government loans or grants, and trade credit. We categorize them into loan, equity and "the rest". In 2011, 36 percent of firms requested at least one type of external financing, about 71 percent of these applied for loans (46 percent for loans only), and 27 percent of businesses who requested financing used "the rest" only. As in 2007, only a very small percentage of firms issued equity in 2011. In non-loan financing, trade credit is used the most.

For those who did not request external financing in 2011, 88 percent of firms reported that external financing was not needed, about 6 percent reported that the request for financing would be turned down or that applying for financing was too difficult (or time-consuming) or the cost of financing was too high.

Table A.1: Percentage of SMEs requesting external financing

Year	Any external financing	Type of requested financing			
		loan at least	equity at least	loan and equity only	the rest only
2011	36	46	1	2	27

In 2011, 25 percent of all firms requested loans (they could also have requested other financing such as trade credit). The mean size of total loan values is slightly higher than $180,000, and the median of total loans is $50,000. In total, about 8 percent of all firms requested trade credit, and 3.7 percent of all firms requested some form of government financing.[19]

In 2011, the majority of external financing was intended to finance the purchase of land and buildings, vehicles, information technology, equipment, and working capital. About 51 percent of firms that requested external financing said this would be used to finance working capital (not necessarily only working capital).

Table A.2: Percentage of loan requests by outcome in 2011

Year	full amount	partial amount	turned down	under review	withdrawn
2011	85	5.0	7.6	-	-

Among those firms that requested loans in 2011, more than 50 percent requested business lines of credit (new or increased limit), 40 percent applied for a business credit card, 35 percent applied for a term loan, and only 16 percent applied for a non-residential mortgage (new or refinancing). Note that these do not sum to 100, as firms may apply for more than one type of loan.

The approval rate for loan applications is high—85 percent of requests were approved for the full amount in 2011, and only 5 percent of requests were authorized for a partial amount. About 7.6

[19] Government financing includes direct loans, loan guarantees, grants, subsidies, and non-repayable contributions and equity from government or government lending institutions.

percent of loan requests were rejected. For those rejected loan requests, the main reasons given for rejection were a poor or lack of credit history and the project was considered too risky. Rejected loans did not appear to be concentrated in one particular type of loan.

About 48 percent of loans used business assets as collateral, 26 percent used personal assets as collateral, and 35 percent did not use any collateral. Note that some loans were refinancing or additions to existing loans, so a firm may report that no collateral was needed for the new part of an existing loan.

Appendix B Score Assigning

We assign scores according to the following criteria. Note: a firm is more likely to be constrained the larger the assigned score is.

1. Not applying for external financing because: (i) thought the request would be turned down, (ii) applying for financing is too difficult or time-consuming, or (iii) the cost of financing is too high.

2. The provider of external financing is the government and/or friends.

3. Loan requests were refused, or approved for a partial amount.

4. Other financing (including leases, equity, trade credit, and government financing) requests were refused or only partially approved.

5. The firm pays a high interest rate relative to other loans of the same type.

To start, we assign the same score to all firms. Then we subtract a score from the initially assigned score depending on how the firm's answers satisfy the above criteria. We assign the highest scores to those that did not request external financing and reported that it is too costly to do so. Firms that requested external financing but do not satisfy any of the above criteria are assigned with the lowest score. In between are firms that satisfy some of the above criteria—for example, loans were partly approved or the firm resorted to the government for loans or loan guarantees. Finally, firms that did not request external financing do not have a score.

The indication of being constrained defined here likely represents a lower bound of the degree of constraint. Firms that made financing requests and obtained full authorization can still be financially constrained. These firms may have knowledge of the underlying financial frictions and, hence, could have taken into account such knowledge and requested an amount that the bank would approve with full authorization. Any higher amount would have been rejected.

In 2011, close to 43 percent of firms needed external financing, including 7.8 percent of firms that needed financing but did not apply for it. Table B.3 reports the percentage of firms by degree of financial constraint. Firms with the highest scores are most likely constrained. Firms reporting no need for external financing have no value for the degree of financial constraint.

Table B.3: Share of firms by likelihood of being financially constrained, 2011

No value	Unlikely	Likely	Most likely
57.0	20.4	13.7	8.9

Appendix C Data Variable Definition

The measured degree of financial constraints is based on the SFSME survey. Most other variables are from the longitudinal administrative data, General Index of Financial Information (GIFI), which is submitted to Canada Revenue Agency when business file a T2 Corporation Income Tax Return or a T5013 Partnership Information Return.

Current Debt: current liabilities, including mainly accounts payable, taxes payable, short-term debt, current deferred income, and current portion of long-term liabilities.

Long-term Debt: includes long-term debt, long-term deferred income,, and future income taxes.

Current Assets: includes cash and deposits, accounts receivable, inventories, short-term investments (e.g. term deposits),

Total Assets: equals current assets plus capital assets (i.e., machinery, equipment, furniture, and buildings) plus long-term financial assets (.e.g, shares, lending).

Cash Flow: equals income before tax but after extraordinary items, plus capital depreciation.

Value Added: equals total sales minus cost of intermediate inputs, the latter equals cost of sales minus wages and crown charges.

Hours worked: is calculated as the annual total payrolls divided average hourly wage by region and at the 3-digit NAICS level. Hourly wage is drawn from Survey of Employment, Payrolls and Hours (SEPH) by Statistics Canada.

Investment: is calculated as year-to-year changes in the net tangible capital assets.

Capital stock: Book value of after-depreciation capital. Alternative capital measure using the perpetual method was also used for robust checks.

When needed, real variables are obtained using the current-price values divided by corresponding implicit prices at the 2-digit NAICS level. These implicit prices are drawn from the multi-factor productivity data sets at Statistics Canada.

Appendix D Financial constraint and investment: an illustrative model

Firm (entrepreneur) j is risk neutral. It enters period t with capital K_{jt} and debt level B_{jt}, chosen in period $t-1$. In order to produce, the firm makes decisions on investment (X_{jt}), future debt (B_{jt+1}), hiring L_{jt}, and intermediate inputs M_{jt}. Loan is the only source of external financing. The production technology is given by

$$Y_{jt} = F(z_{jt}, K_{jt}, L_{jt}, M_{jt}) = e^{z_{jt}+\varepsilon_{jt}} K_{jt}^{\alpha_k} L_{jt}^{\alpha_l} M_{jt}^{\alpha_m}$$

with $\alpha_k+\alpha_l+\alpha_m \leq 1$. Output shifting factor z_{jt} has three components, $z_{jt} = \alpha_a A_{jt}+\omega_{jt}$. Here, A_{jt} is a vector of state variables such as firm age, owner experience, owner age, owner's education attainment. These variables are assumed to be exogenous and could affect output. Productivity shock ω_{jt} follows a Markov process, $\omega_{jt} = h(\omega_{jt-1}) + \eta_{jt}$ with $\eta_{jt} \sim N(0, \sigma_\eta^2)$. The third component ε_{jt} is i.i.d., which can be thought of as the measurement error. The firm does not observe ε_{jt} when making input and output choices.

The firm at the beginning of period t makes decisions on investment, employment, production, and financing. It also decides the optimal dividend payout $D_{it} \geq 0$. The dividend can also be thought of as the consumption of a risk-neutral entrepreneur. The firm's capital stock evolves as $K_{jt+1} = (1-\delta)K_{jt} + X_{jt}$, and the capital adjustment cost is $G(X_{jt}, K_{jt})$. At this point, we only require that function $G(\cdot)$ is convex in both arguments. In q theory of investment, $G(\cdot)$ is homogeneous of degree one with respect to K_{jt}.

D.1 Investment collateral constraint

For now, assume that the decisions on labor and intermediate inputs are static. Define $\tilde{F}(z_{jt}, K_{jt}) = F(z_{jt}, K_{jt}, L_{jt}^*, M_{jt}^*)$ with the starred variables as the optimal choices of labor and intermediate inputs. Wage and material price are implicitly in $\tilde{F}(\cdot)$. The firm's dynamic optimization problem is formed as

$$V(K_{jt}, B_{jt}, A_{jt}, \omega_{jt}) = \max_{\{K_{jt+1}, B_{jt+1}, D_{jt}\}} D_{jt} + \beta E_t V(K_{jt+1}, B_{jt+1}, A_{jt+1}, \omega_{jt+1}) \quad \text{(D.1)}$$

s.t.

$$q_t K_{jt+1} + G(X_{jt}, K_{jt}) + B_{jt} + D_{jt} = \tilde{F}(z_{jt}, K_{jt}, L_{jt}^*, M_{jt}^*) + q_t(1-\delta)K_{jt} + \frac{B_{jt+1}}{R_t}, \quad \text{(D.2)}$$

$$B_{jt+1} \leq \theta q_t K_{jt+1}, \quad \text{(D.3)}$$

$$D_{jt} \geq 0. \quad \text{(D.4)}$$

On the left hand side of the budget constraint is allocation of funds to, in order, next-period capital size, capital adjustment cost, the current debt level, and dividend. The right hand side are the sales of output, value of physical capital, and the future debt level.

The borrowing constraint says that the amount of future debt is proportional to the expected value of physical capital. If $\theta < 1$, the firm is limited to borrow a fraction of the expected value of capital. It follows Kiyotaki and Moore (1997) who rationalize it with the limited enforcement of debt contract. Alternative forms of financial constraints are found in the literature. For example, one can specify a wedge of cost if external fund is used. One can also endogenize θ in the presence of information asymmetry. Karaivanov and Townsend (2014) compare the fit of alternative models of financial constraint.

Buera, Kaboski, and Shin (2011) use a default condition in a limited enforcement contract to determine the upper limit of future capital as $\overline{K}_{jt+1}(K_{jt}, B_{jt}, A_{jt}, \omega_{jt}, \psi)$, where ψ is a parameter of financial friction (degree of contract enforceability) in their model. This limit increases with the firm's productivity and the firm size. The collateral constraint in our model leads to a similar capital limit.

Let λ^k_{jt}, λ^b_{jt}, and λ^d_{jt} be the Lagrangian multiplier respectively for the firm's budget constraint, borrowing constraint, and dividend. The firm's first-order necessary conditions are as follows

$$D_{jt}: \; 1 - \lambda^k_{jt} + \lambda^d_{jt} = 0; \tag{D.5}$$

$$K_{jt+1}: \; \lambda^k_{jt}\left[-q_t - G_1(X_{jt}, K_{jt})\right] + \lambda^b_{jt}\theta q_t + \beta E_t\left[\lambda^k_{jt+1}\Delta_{jt+1}\right] = 0; \tag{D.6}$$

$$B_{jt+1}: \; \lambda^k_{jt}/R_t - \lambda^b_{jt} - \beta E_t\left[\lambda^k_{jt+1}\right] = 0. \tag{D.7}$$

Here, $\Delta_{jt+1} = \tilde{F}_K(z_{jt+1}, K_{jt+1}) + q_{t+1}(1-\delta) - G_2(X_{jt+1}, K_{jt+1})$. It can be seen that the Euler equation for K_{jt+1} is

$$q_t + G_1(X_{jt}, K_{jt}) - \frac{\lambda^b_{jt}}{\lambda^k_{jt}}\theta q_t = \beta E_t \frac{\lambda^k_{jt+1}}{\lambda^k_{jt}}\Delta_{jt+1}.$$

Non-binding financial constraint. If the borrowing constraint does not bind, i.e. $\lambda^b_{jt} = 0$, we then have the standard inter-temporal condition for the optimal investment

$$q_t + G_1(X_{jt}, K_{jt}) = \frac{1}{R_t} E_t \Delta_{jt+1}. \tag{D.8}$$

In this case, the expected marginal gains of investment, $E_t\Delta_{jt+1}$, equals the marginal cost of investment $q_t + G_1(X_{jt}, K_{jt})$. Note that the expectation E_t is conditional on current state ω_{jt}, so the expected future marginal gains of investment is also dependent upon ω_{jt}. The optimal choice K_{jt+1} therefore depends on the current productivity level ω_{jt} because of serially correlation of productivity. The higher the current productivity level, the larger the future gain of investment, and this positive relation depends on the degree of persistence of productivity. **The optimal investment does not depend on the current net worth, although they are correlated through the total factor productivity ω_{jt}.** Let K^*_{jt+1} the optimal unconstrained future capital implied in Equation D.8. The demand for loan in period t is

$$B^*_{jt+1} = q_t K^*_{jt+1} + G(X^*_{it}, K_{it}) + B_{jt} + D_{jt} - \tilde{F}(z_{jt}, K_{jt}) - q_t(1-\delta)K_{jt}.$$

If the borrowing constraint is not binding, we have $\frac{\partial B_{jt+1}}{\partial K_{jt}} = q_t \frac{\partial K^*_{jt+1}}{\partial K_{jt}} + G_2(X^*_{it}, K_{it}) - \tilde{F}_k(z_{jt}, K_{jt}) - q_t(1-\delta)$. By assumption, $G_2(X^*_{it}, K_{it}) < 0$.[20] Demand for external financing can decline as firms becomes

[20] $G_2(\cdot)$ can be positive if $X^*_{jt} < 0$, but then the demand for borrowing will be smaller than B_{jt}.

larger.

Binding financial constraint. If the borrowing constraint binds, then $B_{jt+1} = \theta q_t K_{jt+1}$. Let the firm's realized net worth in period t be $N_{jt} = \tilde{F}(z_{jt}, K_{jt}) + q_t(1-\delta)K_{jt} - B_{jt} - D_{jt}$. Budget constrain becomes

$$q_t K_{jt+1} + G(X_{jt}, K_{jt}) = N_{jt} + \frac{1}{R_t}\theta q_t K_{jt+1}, \tag{D.9}$$

from which we can solve for K_{jt+1}. Clearly, investment is bounded by the firm's net worth plus the amount the firm can borrow.

D.2 Working capital financial constraint

We derive the first-order conditions with respect to labor choice when the firm borrows to finance the wage bill. Assume that the firm borrow to finance a fraction κ of wage payment, for which the firm need to pay an interest rate. Abstract from the intermediate inputs, and re-write the firm's dynamic problem as follows

$$V(K_{jt}, B_{jt}, A_{jt}, \omega_{jt}) = \max_{\{K_{jt+1}, B_{jt+1}, D_{jt}\}} D_{jt} + \beta E_t V(K_{jt+1}, B_{jt+1}, A_{jt+1}, \omega_{jt+1})$$

s.t.

$$q_t K_{jt+1} + G(X_{jt}, K_{jt}) + B_{jt} + D_{jt} + w_t L_{jt} + (R_t - 1)\kappa w_t L_{jt} = F(z_{jt}, K_{jt}, L_{jt}) + q_t(1-\delta)K_{jt} + \frac{B_{jt+1}}{R_t},$$

$$B_{jt+1} + \kappa w_t L_{jt} \leq \theta q_t K_{jt+1},$$

$$D_{jt} \geq 0.$$

Again, let λ^b_{jt} be the Lagrangian multiplier for the collateral constraint. The first-order condition for labor choice is

$$F_L(z_{jt}, K_{jt}, L_{jt}) = [1 + (R_t - 1 + \lambda^b_{jt})\kappa] w_t. \tag{D.10}$$