A Model of Costly Capital Reallocation and Aggregate Productivity*

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Abstract

This paper studies the effects of capital reallocation on aggregate productivity. Capital reallocation (i.e., the flow of productive capital across firms through changes in ownership) is an important activity in the United States: on average, the total value of capital reallocation annually is 3 to 4 percent of U.S. GDP. Firms with lower productivity are more likely to be reallocated to more-productive firms, and they also experience increases in productivity after reallocation. I use a dynamic model of firm investment and capital reallocation to match these facts. In the model, increased participation in the acquisition market by heterogeneous firms results in an increase in aggregate labor productivity, while the productivity of capital declines due to decreasing returns to scale. With the calibrated model, I show that the increased reallocation of capital contributed 20 percent of the growth of aggregate labor productivity during the period 1985-1990, and 22 percent during the period 1995-2000.

JEL Codes: E22, L16

1 Introduction

Empirical studies show that the reallocation of production inputs across firms and establishments is an important source of change in aggregate productivity. Much of aggregate productivity growth can be attributed to reallocation.[1] Capital reallocation through change in asset ownership across firms and establishments is an important component of reallocation and is the way in which many firms expand or downsize.[2] This reallocation has been increasing since the 1960s in the United States, reaching a record high in 1999, at 15 percent of the GDP. Although previous empirical studies have estimated the productivity gains from reallocation at the firm level, the importance of such reallocation in the growth of aggregate productivity has not been investigated. On the other hand,

Most macroeconomic models typically assign no role to capital reallocation across firms or implicitly assume that capital can be costlessly reallocated from one firm to the other. This assumption, however, is quite strong. First, it is at odds with the data. For instance, firms in the U.S. do acquire capital from (or sell capital to) other firms as a way to expand (downsize). In fact, since the 1960's capital reallocation through changes in ownership has averaged 3 to 4 percent of GDP per annum reaching a high of 15 per cent of GDP in 1999 (see Holmstrom and Kaplan, 2001). Second, there is evidence that it may be costly to transfer capital from one firm to the other. For instance, Ramey and Shapiro (2001) use equipment transaction data of aerospace plants and find that, due to sectoral specificities, physical capital sells for a substantially discounted price relative to replacement cost. Likewise, Kurmann and Petrosky-Nadeau (2007) find evidence of costly capital reallocation due to search and matching frictions in the capital goods market while Eisfeldt and Rampini (2007) find that financially constrained firms are more likely to purchase used capital. To the extent that there is a role for capital reallocation in the real world but frictions or costs of reallocation limit the flow of capital from less productive firms to more productive firms, this will have implications for aggregate productivity. The objective of this paper is to study the quantitative implications of costly reallocation of capital on aggregate productivity.

To this end, the paper develops a general equilibrium model with heterogeneous firms that explic-

[1] See Bartelsman and Doms (2000) for a survey.

[2] Empirical studies that decompose aggregate productivity growth generally consider between-firm growth and net entry as reallocation, (see Foster et al., 2001). In this paper, we define reallocation explicitly as the process of one firm buying (selling) physical capital from (to) another firm.

itly accounts for the costly reallocation[3] of capital. The model builds on Jovanovic and Rousseau (2002) which proposes a q-theory of mergers to explain why less productive firms are more likely to be taken over by more productive firms. In our model, heterogeneous firms use physical capital and labour to produce with a technology featuring decreasing returns to scale and are subject to idiosyncratic productivity shocks. Firms can invest either by purchasing new capital or by purchasing used capital from other firms in the used-capital (or acquisition) market. There are however different adjustment costs to invest in new or used capital. We show that due to the different adjustment costs, it is optimal for firms to participate in the used-capital market (in the event of which capital is reallocated to them) only when their total investment demand surpasses a certain threshold value. We calibrate our model to match firm-level data on investment and reallocation for the period of 1981 to 2004 in the United States. We then use our calibrated model to conduct two exercises.

In the first exercise, we assess the importance of capital reallocation as a source of aggregate productivity. We do this by comparing aggregate productivity in a world with and without the used capital market.We find that relative to the benchmark calibration where the used capital market operates, when the used capital market is shut down due to high used capital adjustment costs, aggregate output decreases by 2.2 percent and aggregate labor productivity drops by 3 percent. The decrease in aggregate labor productivity arises because when capital reallocation is too costly, the demand for investment falls, leading firms to downsize and reduce their output. In addition, firms substitute labor for capital. Thus, the ability to reallocate capital through the used capital market helps to increase labor productivity and, in our model, this channel is quantitatively important.

We then assess the extent to which the ability to reallocate capital may have contributed to increases in aggregate labor productivity observed in the U.S. data on industry productivity. We focus on two empirical episodes where capital reallocation grew faster than average: (i) 1985 to 1990 where an increase in the investment rate in used capital of 3 percentage points was accompanied by an increase in aggregate labor productivity of 9 per cent, and (ii) 1995 to 2000 where an increase in the investment rate in used capital of 7 percentage points was accompanied by an increase in aggregate labor productivity of 18 percent. We conduct our experiments by reducing the adjustment cost of used capital from our benchmark

[3] In this paper, we define reallocation as the process of one firm buying (selling) physical capital from (to) another firm. There are alternative definitions of reallocation, see for example Foster et al. (2001).

calibration so that the model predicts the same increase in capital reallocation as shown in the data over each respective period. We then derive the implications of these changes in the cost of reallocation on aggregate labor productivity. We find that the increased capital reallocation accounts for 20 percent of the growth in labor productivity from 1986 to 1991, and 22 percent from 1996 to 2001. Further, the source of the increase in labor productivity is predominantly the increase in output rather than a decrease in labor input.

The paper proceeds as follows. Section 2 discusses the facts on capital reallocation, as well as the related literature. Section 3 introduces the model. Section 4 conducts equilibrium analysis. Section 5 calibrates the model to the data regarding investment and reallocation. Section 6 assesses the implications of the costly reallocation of capital, and Section 7 concludes.

2 Related Literature

The modeling of reallocation in this paper is consistent with empirical studies using U.S. establishment-level data. These studies find that less productive firms are more likely to be reallocated and reallocated firms on average experience productivity improvement, see McGuckin and Nguyen (1995), Lichtenberg and Siegel (1989), Lichtenberg (1992), Maksimovic and Phillips (2001), and Schoar (2002), among others.[4] Lichtenberg and Siegel (1989) use data on large U.S. manufacturing plants from the Longitudinal Research Data set (LRD) to study the effects of changes in ownership on productivity. The authors find that plants with lower total factor productivity (TFP) are more likely to be sold, and on average plant-level TFP increases by 23 percent for reallocated plants. McGuckin and Nguyen (1995) also use the LRD and find that the acquired plants tend to have lower productivity. Moreover, plants that experience changes in ownership gain productivity during the 5 to 9 years following these changes. Maksimovic and Phillips (2001) extend these studies by looking into more broad ownership changes that include partial establishment sales in LRD from 1974 to 1992. They find that, on average, 3.9 percent of establishments change ownership annually. Reallocated plants or whole firms experience significant gains in productivity. The buyers tend to have higher TFP and tend to be larger than the sellers.

That the buyers are more productive than the sellers is also shown in other papers. Jovanovic and Rousseau

[4]An exception is Ravenscraft and Scherer (1989) who find evidence that supports a decline in post-merger productivity declines in 1960s.

(2002) find that in nearly 70 percent of mergers and acquisitions in the United States the buyers have a larger Tobin's Q value than the sellers. Their estimation shows that Tobin's Q value and the investment in used capital are significantly and positively correlated. Erard and Schaller (2002) obtain similar findings.

Our paper is related to the literature on used capital in that reallocation refers to the change in ownership of used capital. Eisfeldt and Rampini (2007) find that financially constrained firms (often small and young) are more likely to purchase used capital goods than unconstrained firms. Pulvino (1998), on the other hand, finds that financially constrained airlines receive lower price offers for commercial aircraft than unconstrained firms, while Ramey and Shapiro (2001) emphasize that the sectoral specificity of physical capital may cause the sale price of used capital to be low. Our model is related to used capital in that the capital purchased by one firm from another is indeed used. However, it will be clear later that our model emphasizes the lumpiness of investment in used capital, which is closer to acquisition transactions.

The model is closely related to two papers. Jovanovic and Rousseau (2004) use technology adoption to explain the waves of mergers in the United States. When a new technology is invented, firms can either adopt it or choose to exit through acquisitions. Our model also examines the two recent merger waves but assumes that increased merger activity arises from the lower cost of reallocation. Further, the goal of our model is not to explain the merger waves but to study their implications for aggregate productivity. Eisfeldt and Rampini (2006) find that the procyclical reallocation of capital can be explained by the cost of counter-cyclical reallocation. Our model also relies on cost of reallocation to generate increased reallocation, but studies the stationary equilibrium. Neither of the two papers studies the implications of reallocation for aggregate productivity.

3 Model

The model economy consists of one representative household and a continuum of firms, split between incumbents and new entrants. Each period, the representative household makes decisions on consumption, saving, and labor supply. On the other hand, at the beginning of each period, incumbent firms face an exogenous probability of exit from the industry. Conditional on their survival, they make decisions on investment in physical capital and employment given the idiosyncratic shocks to productivity that they face. Potential entrants receive a signal about their productivity and decide whether to enter the industry or not.

We assume that each firm is operated by a manager who maximizes the firm's profit.[5] We denote the firm's productivity by ε, which may reflect managerial ability and level of technological efficiency. Given the same amount of capital, k, and labor, l, firms with higher ε produce more and invest more in capital. The production function is $f(k,l,\varepsilon) = z\varepsilon(k^{1-\alpha}l^{\alpha})^{\nu}$, where $\alpha \in (0,1)$ and $\nu \in (0,1)$. The decreasing returns to scale in capital and labor imply that the firm size is finite and bounded from above and zero. The aggregate technology level, z, is constant, and is used for comparative statics in later sections.

Firm-specific productivity is assumed to follow an AR(1) process, $\log\varepsilon_t = \rho_\varepsilon \log\varepsilon_{t-1} + \eta_t$, where ρ_ε is the coefficient of serial correlation. The innovation term, η_t, is independent and identically distributed, with $\eta_t \sim N(0,\sigma_\eta^2)$.

Adjusting labor does not incur any cost, so the firm's static labor choice can be obtained as

$$l^* = \left(\frac{\alpha\nu z\varepsilon}{w}\right)^{\frac{1}{1-\alpha\nu}} k^{\frac{(1-\alpha)\nu}{1-\alpha\nu}},$$

where w is the wage rate. The firm's revenue net of labor cost is then given by

$$R(\varepsilon,k) = \frac{1-\alpha\nu}{\alpha\nu} w l^*.$$

3.1 Investment and reallocation

In each period, firms that survive choose the optimal investment in physical capital for the next period. Capital can be purchased from the new capital market and from other firms through the acquisition market. We follow Eisfeldt and Rampini (2006) and call capital purchased from other firms used capital. Both types of capital are equally productive. The key difference, however, is that used capital purchased in the acquisition market is already assembled by the selling firm and is ready for production. Given that used capital is already assembled, it is at the outset ready for production. Therefore, in this paper, we assume that it is less costly to the firm to adjust total capital stock with used capital than new capital.

More formally, let x_n denote investment in new capital and x_u denote investment in used capital.

[5] We abstract from organizational or agency problems related to managerial issues.

Following Jovanovic and Rousseau (2002), if the firm buys new capital, the adjustment cost of capital is $h_1(x_n, k)$ where h_1 is convex in x_n. If the firm buys used capital in the acquisition market, it pays p_u per unit of capital. The adjustment cost of used capital is $h_2(x_u, k)$ where h_2 is convex in x_u. We make the following assumption on convex adjustment costs:

Assumption 1. $\frac{\partial h_1(x_n,k)}{\partial x_n}|_{x_n=x} > \frac{\partial h_2(x_u,k)}{\partial x_u}|_{x_u=x}$.

Thus, for a given investment level x, the cost of an additional unit of investment in new capital is higher than the cost of an additional unit of investment in used capital. This assumption implies that, other things equal, a firm would prefer to buy or sell capital in the acquisition market.

However, we assume that participating in the used capital market is costly. Those participation costs can arise, for instance, due to search or information frictions or they can arise from transaction costs related to acquiring a firm. Specifically, we assume that firms need to pay a fixed cost, Φk, to participate in the acquisition market. Thus the fixed cost increases with the firm's capital size, capturing the idea that a firm with large capital stock will have to forego more output because of disruption. This fixed cost also implies that small firms can participate in the acquisition market if their productivity shock is large enough.[6]

It is worthwhile to note that even though the adjustment cost for new capital is higher than that for used capital, it is *not* optimal for the firm to invest solely in used capital once it has paid the fixed cost for participating in the used capital market. Letting $i = x/k$ denote the total investment rate, in Appendix B we prove the following proposition

Proposition 1. *If adjustment costs for new and used capital are* $h_1(x_n, k)$ *and* $h_2(x_u, k)$*, respectively, then choosing* $x_n = 0$ *is never optimal when* $i \neq 0$.

[Figure 1 here.]

The intuition for this result lies in the convexity of the adjustment cost profiles and is illustrated in Figure 1. In the figure, for any given total investment level, the blue dashed line depicts a more convex

[6] There are alternative approaches to modeling frictions to investment in the literature, e.g., firm's financial constraint, search frictions, and informational frictions in the used-capital market, see Kaplan (2000), House and Leahy (2004), Eisfeldt (2004) and Kurmann and Petrosky-Nadeau (2007) among others. In addition, in reality, some acquisitions fail, meaning that productivity after acquisition does not increase. Our model is consistent with this fact in that in our model buyers of used capital are subject to idiosyncratic shocks, facing the risk of a productivity decline after purchasing used capital.

adjustment cost profile for new capital than that for used capital (green solid line). Suppose that the firm's total investment level is 0.6 and the firm only invests in used capital. Then, the firm would bear an adjustment cost equal to A on the vertical axis. Now, suppose that the firm switches 0.1 of the total investment to investment in new capital such that $x_n = 0.1$ and $x_u = 0.5$. The firm would then face an adjustment cost equal to $B + C < A$. Thus the firm can save on the total adjustment cost that it would otherwise save by investing some amount in new capital. It should be noted that in Figure 1 we set $p_u = 1$. For other values of p_u, in Appendix B, we show that if the firm invests at all, it is always optimal to invest some amount in new capital.

Optimal split between new and used capital

Now that we have established that the firm would invest some amount in new capital, we are in a position to determine the optimal split between new and used capital. Assembling new capital and training workers to use it is more costly than buying ready-to-produce used capital. However, because of the fixed cost of acquiring used capital, firms would rather buy new capital when their demand for investment is low. As the demand for investment increases above some threshold level, the firm will switch some investment to used capital to minimize the total adjustment cost.

To show why this is the case, let the investment rate in used capital be $i_u = x_u/k$. Given a total investment level x, the firm's choice of splitting x between x_u and x_n is determined in the following static cost-minimization problem

$$\min_{i_u}\{\tilde{i} - i_u + p_u i_u + \frac{\gamma_n}{2}(\tilde{i} - i_u)^2 + \frac{\gamma_u}{2} i_u^2 + \Phi\}.$$

Suppose that $h_1(x_n, k) = \frac{\gamma_n}{2}\frac{x_n^2}{k}$ and $h_2(x_u, k) = \frac{\gamma_u}{2}\frac{x_u^2}{k}$ with $0 < \gamma_u < \gamma_n$.[7] If the firm chooses $i_u \neq 0$, solving the above cost minimization problem gives the optimal choice of i_u, as $i_u^* = \frac{\gamma_n}{\gamma_n+\gamma_u} i + \frac{1-p_u}{\gamma_n+\gamma_u}$. This optimal investment rate in used capital is a linear function of total investment rate.

Next, let $\tilde{i}$ be the investment rate at which the firm is indifferent between choosing $i_u = 0$ and $i_u \neq 0$. At $\tilde{i}$, the unit cost of investing in new capital equals the unit cost of investing in both types of capital, as

[7] Note that, without loss of generality, we will assume quadratic costs of adjustment for the remainder of the paper.

follows

$$\widetilde{i} + \frac{\gamma_n}{2}(\widetilde{i})^2 = \widetilde{i} - i_u^* + p_u i_u^* + \frac{\gamma_n}{2}(\widetilde{i} - i_u^*)^2 + \frac{\gamma_u}{2}(i_u^*)^2 + \Phi,$$

where we have plugged i_u^* into the cost of investing in both types of capital (on the right hand of the equation). Solving the above equation, we can obtain the threshold investment rate $\widetilde{i}$ at which the firm is indifferent between investing only in new capital and investing in both types of capital.

Figure 2 shows an example of the optimal split between investments in new and used capital. The horizontal axis is investment rate, and vertical axis are investment levels for new and used capital. The dashed red line shows the optimal rate of investment in used capital. When the adjustment of capital from the current capital stock is small, falling between $\widetilde{i}_1$ and $\widetilde{i}_2$, the firm does not participate in the used-capital market, setting the investment in used capital to zero. As the adjustment (selling or buying) of capital from the current level becomes larger, the firm reduces investment in new capital, and increases that in used capital.

[Figure 2 here.]

3.2 Incumbents

At the beginning of each period, firms first draw the probability of exit s before observing the realization of productivity shock and before any decision is made. If drawn to exit, with probability $1-\xi$ the capital scrap value of an exiting firm is zero, and with probability ξ an exiting firm can sell capital in the acquisition market after paying the adjustment cost $\left(\frac{\gamma_u}{2} + \Phi\right) k$. Hence the exit value is $V^x(k) = \xi\left(p_u - \frac{\gamma_u}{2} - \Phi\right) k$. The exit probability and the probability of selling capital in the acquisition market upon exit are the same for all firms. The random exit assumption has been used in previous literature, for example, Restuccia and Rogerson (2008) as a parsimonious simplification of a complex process.

Let $V^c(\varepsilon, k)$ be the continuation value, the incumbent's optimal problem is given by

$$\begin{aligned} V^c(\varepsilon, k) &= \max_{\{x_n, x_u\}} R(\varepsilon, k) - x_n - p_u x_u - h_1(x_n, k) - [h_2(x_u, k) + \Phi k] \cdot 1_{\{x_u \neq 0\}} \\ &\quad + \frac{1}{1+r}\left[(1-s) E V^c(\varepsilon', k') + s V^x(k')\right] \end{aligned} \tag{1}$$

subject to $k' = (1-\delta)k + x_n + x_u$. The cost for the used capital is zero if $x_u = 0$. The gross interest rate

$(1 + r)$ is endogenously determined in equilibrium.

3.3 Entrants

A continuum of potential entrants exists and decides whether or not to enter the industry. At the beginning of each period, a potential entrant receives a signal q on its productivity at entry, drawn from the cumulative distribution $\Omega(q)$. Lee and Mukoyama (2010) interpret the productivity signal q as ideas; a good idea would lead to high post-entry productivity. Observing this signal, the potential entrant makes the entry decision. If entering, it chooses the optimal start-up investment. After entry the firm's draw of productivity shock follows the conditional distribution $\pi_e(\varepsilon'|q)$, and from then on the entrant becomes an incumbent. An entrant receiving signal q solves for the following problem:

$$V^e(q) = \max_{k'} \int_{\varepsilon'} \frac{1}{1+r} V^c(\varepsilon', k') \pi_e(d\varepsilon'|q) - k' - \psi k'. \tag{2}$$

The potential entrant chooses capital k' for the next period by maximizing the expected value in the first period of production net of entry cost $\psi k'$. Entry happens if $V^e(q) \geq 0$. We assume that the conditional distribution of productivity for new firms is increasing in q, which implies that there exists a threshold value of the signal $\widetilde{q}$ at which $V^e(\widetilde{q}) = 0$. Firms enter only if their signal satisfies $q \geq \widetilde{q}$. Entering firms are different in their capital size at entry, $k' = g_e(q)$.

For computational simplicity, we assume that entrants buy capital from the new capital market. Otherwise, the relative price of used capital p_u would be present in the entry cost, complicating computations. In addition, we do not have data on the fraction of used capital investment for entrants. Nevertheless, in our model small and young firms do participate in the acquisition market, as is consistent with the data.

The *ex ante* expected value of entry (before drawing signal q) is zero under the free-entry condition. Let the *ex ante* entry cost be c_f. The free entry condition implies the following

$$\int_{q \geq \widetilde{q}} V^e(q) d\Omega(q) - c_f \leq 0. \tag{3}$$

[New august 2013]. Entrants draw a productivity signal q, pay fixed cost c_f, then decide to enter

$$\int_{q \geq \tilde{q}} V^e(q) d\Omega(q) - c_f \leq 0, \tag{4}$$

where $V^e(q) = \int_{\varepsilon'} \frac{1}{1+r} V^c(\varepsilon', k') \pi_e(d\varepsilon'|q)$.

3.4 Household

The economy has one representative household with preference

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \theta L_t],$$

where c_t is consumption and L_t is the fraction of individuals that are employed. This preference is used in Hansen (1985) and Rogerson (1988). The household owns all firms. Since there is no aggregate uncertainty, the household's optimization is deterministic. In each period, the household chooses the optimal consumption c, the labor supply L, and saving A'. Let w be the wage rate relative to the output price, the household's recursive optimization problem is

$$W(A) = \max_{c,L,A'} \left[u(c) - \theta L + \beta W(A') \right], \tag{5}$$

subject to

$$c + A' \leq wL + (1+r)A + \Pi,$$

where Π is the total profit of firms (after all capital cost and labor cost). From the first-order conditions, we have that $u'(c) = \frac{\theta}{w}$ and $(1+r) = \beta^{-1}$.

4 Equilibrium Analysis

Our focus is on the stationary equilibrium with positive entry and exit. We first characterize the firm distribution and examine the market clearing conditions, then define the stationary equilibrium.

4.1 Firm distribution

Let the firm distribution be $\mu(\varepsilon, k)$, and $\pi(\varepsilon'|\varepsilon)$ be the transition probability of the productivity shock ε. Further, let the incumbent's policy function be $k' = g(\varepsilon, k) \in \mathscr{K}$ if it stays and $k' = 0$ if it exits. The transition matrix for the firm's state is given by

$$P(\varepsilon', k'|\varepsilon, k) = \begin{cases} 1_{\{k'=g(\varepsilon,k)\}} \cdot \pi(\varepsilon'|\varepsilon), & \text{if stays,} \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Recall that the entrant's policy function is $k' = g_e(q)$. The transition matrix for the potential entrant receiving signal q is given by

$$P^e(\varepsilon', k'|q) = \begin{cases} 1_{\{k'=g_e(q)\}} \cdot \pi_e(\varepsilon'|q), & \text{if } V^e(q) > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

The firm distribution over (ε, k) is summarized by the probability measure μ defined on $\mathbb{S}$, where $\mathbb{S}$ is the σ-algebra generated by the open subsets of the product space $(\mathscr{E}, \mathscr{K})$ on the non-negative real line. Let M be the mass of new entrants. The distribution of continuing firms is $\mu^c(\varepsilon, k) = \mu(\varepsilon, k) \cdot (1-s)$. The evolution of the firm distribution is given by

$$\mu(\varepsilon', k') = \int_{\mathbb{S}} P(\varepsilon', k'|\varepsilon, k) d\mu^c(\varepsilon, k) + M \int_{q \geq \tilde{q}} P^e(\varepsilon', k'|q) d\Omega(q). \tag{8}$$

On the right hand side of Equation (8), the first term is the distribution of firms that stay in the industry, the second term is the distribution of entrants on $\mathbb{S}$. Using the matrix form, the firm distribution can be written as

$$\mu' = [\mu \cdot (1-s)]^T P + M\Omega^T P^e,$$

where μ and $\Omega^T P^e$ are distributions over the state (ε, k) for incumbents and entrants, respectively. If the invariant distribution exists, then we have

$$\mu = [I - (1-s) \cdot P]^{-1} \cdot [M\Omega^T P^e], \tag{9}$$

where I is the identity matrix. The timing of this process is as follows: at the beginning of period t, the firm distribution over (ε, k) is μ_t. Of this distribution, some firms exit before commencing the production. The entrants enter at the beginning of period t after observing their own signal q_t. The period-t entrants then draw their productivity shock from the conditional distribution $\pi_e(\varepsilon_{t+1}|q_t)$.

It should be noted that the invariant distribution function is proportional to the entrant mass M, a property to be used for solving the equilibrium.

4.2 Output market

Let $u(c) = \log c$. From the household's problem, the aggregate consumption satisfies $c = \frac{w}{\theta}$. Not surprisingly for the quasi-linear preference, the optimal aggregate consumption is independent of the non-labor income.

In the output market, we can ignore used capital because in equilibrium the aggregate net investment in used capital is zero due to the market clearing condition. The acquisition market affects output and consumption only through the used capital adjustment cost and participation cost. The net aggregate output that the household can use for consumption in each period is computed as the weighted output level of all firms net of costs related to capital. In the stationary equilibrium, the continuing firm has the following real revenue:

$$\Pi(\varepsilon,k) = z\varepsilon(k^{1-\alpha}l^{\alpha})^{\nu} - x_n(\varepsilon,k) - h_1(x_n(\varepsilon,k),k) - [h_2(x_u(\varepsilon,k),k) + \Phi k]\cdot 1_{\{x_u\neq 0\}}.$$

It should be noted that new capital is produced as part of output in the model. One unit of output can be transformed into one unit of new investment good.

Exiting firms with capital size k pay the adjustment cost $(\frac{\gamma_u}{2}+\Phi)k$ if their capital is sold in the acquisition market. Entrants pay the start-up costs $(k' + \psi k')$ but do not produce in the entry period. Putting all outputs and costs together, the net aggregate output used for consumption is as follows

$$\begin{aligned} C^s(\mu) \quad = \quad & (1-s)\int_{\mathbb{S}} \Pi(\varepsilon,k) 1_{\{g(\varepsilon,k)\in\mathcal{K}\}} d\mu(\varepsilon,k) - s\xi \int_{\mathbb{S}} (\frac{\gamma_u}{2}+\Phi) k d\mu(\varepsilon,k) \\ & -M\int_{q>\tilde{q}} (1+\psi) g_e(q) d\Omega(q) - Mc_f. \end{aligned} \tag{10}$$

The first term on the right-hand side is the aggregate output of continuing firms, the second term is the participation cost in the acquisition market by the exited firms, and the last two terms are the entry cost and investment by entrants.

The aggregate supply of output for consumption is proportional to the entry mass M (recall that $\mu(\varepsilon, k)$ is proportional to M). Later, this property is used for solving for M.

4.3 Used capital market

Appendix B shows that there exists a threshold value of investment rate $\widetilde{i}$ at which the firm is indifferent between choosing $x_u = 0$ and $x_u \neq 0$. Given the price p_u, if the firm participates in the acquisition market, we can use the firm's optimal investment splitting rules to obtain that a firm in the state (ε, k) buys used capital if its total investment rate $i(\varepsilon, k) > \widetilde{i}_2$, and it sells used capital if $i(\varepsilon, k) < \widetilde{i}_1$. As the price p_u increases, firms tend to buy less or sell more used capital.

The aggregate supply of used capital is

$$\Lambda(p_u) = (1-s)\int_{\mathbb{S}} 1_{\{x_u(\varepsilon,k)<0\}} x_u(\varepsilon, k)\, d\mu(\varepsilon, k) + s\xi \int_{\mathbb{S}} k\, d\mu(\varepsilon, k).$$

$\Lambda(p_u)$ is negative and a decreasing function of price p_u. The aggregate demand for used capital is

$$\Psi(p_u) = (1-s)\int_{\mathbb{S}} 1_{\{x_u(\varepsilon,k)>0\}} x_u(\varepsilon, k)\, d\mu(\varepsilon, k),$$

which is positive and decreasing in p_u. In equilibrium, the two measures sum to zero.

4.4 Recursive equilibrium

We are now in a position to define the stationary recursive equilibrium, as a set of functions,

$$\left(w, p_u, M, V, W, L^d, L^s, A', K', c, \mu\right),$$

such that the household and firms maximize their expected values, and the markets for reallocation, labor, and output clear, as follows

(i) Given prices, V solves the firm's Bellman equation (1), and $l(\varepsilon, k, \mu)$ and $g(\varepsilon, k, \mu)$ are the firm's policy functions for labor and capital, for all $(\varepsilon, k) \in \mathbb{S}$.

(ii) Given prices, W satisfies the household's problem (5), (c, L^s, A') are the associated policy functions, and $c = \frac{w}{\theta}$.

(iii) The used capital market clears such that p_u solves the equation $\Psi(p_u) + \Lambda(p_u) = 0$.

(iv) The asset market clears: $A' = K'$, where the aggregate future capital is $K'(\mu) = \int_{\mu \in \mathbb{S}} g(\varepsilon, k, \mu) d\mu$.

(v) The output market clears: $c = C^s(\mu)$, where $C^s(\mu)$ is given by Equation (10).

(vi) The free-entry condition (3) is satisfied.

(vii) The firm distribution is given by Equation (9).

The definition of the stationary equilibrium is similar to that in Hopenhayn and Rogerson (1993). The difference lies in the capital markets and non-degenerate capital size distribution of entrants in our model. The wage rate is proportional to M, according to equilibrium conditions (ii) and (v). The stationary equilibrium is solved by iterating a two-step loop. In the first step, given a guessed entry mass M we use equilibrium conditions (iii) and (vi) to solve for p_u and w. The excess demand for the used capital does not depend on the entry mass M, so p_u can be obtained for any value of M. The equilibrium wage w is obtained from the free-entry condition, as a function of M. In the second step, given p_u and w obtained from the first step, we obtain the entry mass M from the output market clearing condition (v). These two steps are iterated until entry mass M converges.

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5 Data

The data on investment and reallocation are from Compustat 1975-2007. The sale of property, plant and equipment (PPE) was available only since 1975. For data moments, we use the manufacturing sector for the period between 1981 and 2004. Selecting this sub-period is because acquisition has a time trend and the model experiments focus on merger waves in mid-1980s and late 1990s. Moment alues only change slightly if the mineral industries and construction industries are included.

Table 5 reports data moments regarding investment and acquisitions. Slightly more than 21 percent of firms/year observations saw positive capital acquisitions, among which the average acquisition capital ratio is about 13.6 percent.

In principle, reallocation can consist of both acquisitions and capital sales. In this paper we focus on acquisitions, instead of both. This is primarily due to that we do not explicitly model both a positive investment and positive capital sales, while in the data we typically observe both. In the Compustat sample used for data moments, about 55 percent of observations saw positive capital sales. Among the observations with positive capital sales, only 3.4 percent of them saw less than 1 percent of investment capital, and no firms had zero investment. This suggests that half of firms in a typical year bought (likely new) capital and sold (or retire) old capital, possibly to upgrade the capital quality. Since our model abstracts from capital vintage, investment in the model is the net change of capital stock. Lastly, capital sale in Compustat includes both sale and retirement of capital, but the two ways of shrinking capital stock have rather different implications for modeling capital reallocation.

, instead of taking as reallocation the sum of acquisition and capital sale. The main reason is that our model concentrates on the lumpiness of investment in used capital which resembles more like an acquisition than small amount of capital sale. In addition, data show that acquisition and capital sale should be modeled differently. In Compustat 1981-2004, on average, only 27 percent of firm/year sample has positive acquisition while 42 percent of firm/year sample has positive capital sale. Acquisition indeed as we model it displays a large lumpiness. Conditional on acquiring capital, the average acquisition to capital stock ratio is 30 percent; while conditional on selling capital, the average capital sale to capital stock ratio is only 2.7 percent.

Table 5 reports the investment and reallocation moments for manufacturing industries. In this table, firm-level moments are computed as the averages of firm-level values. Investment rate is the average of firm investment rates. Spike rate is the proportion of firm-year observations with an investment rate larger than 20 percent. Inaction is the proportion of firm/year observations with an investment rate less than 1 percent. Participation rate is the proportion of firm-year observations that have non-zero acquired capital value. Aggregate moments are computed as the averages of annual aggregate moments. For example, the aggregate reallocation rate (acquisition capital ratio) in each year is calculated as the total value of acquired capital divided by the total value of capital in the entire industry. We then take the average of this ratio over the period 1981-2004. Firm-level moments are the average of the same moment by individual firms. In the case of reallocation (acquisition), firms not acquiring capital from other firms are considered to have zero acquisition and are counted in calculating the firm-level reallocation rate.

[Table 5 here.]

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6 Model Calibration

6.1 Parameter values

We choose parameter values for preference and production to target the moments of investment and reallocation for U.S. firms. Table 1 summarizes the calibrated parameter values and targets. The discount factor is set to $\beta = 0.96$, to match an annual real interest rate of 4 percent. The share of leisure in utility is set to $\theta = 0.94$, so that about 80 percent of the population participates in labor market. The decreasing returns to scale of the firm's production function is set as $\nu = 0.896$, approximately the average of its various estimates in Lee (2005). The Cobb-Douglas technology parameter is set to $\alpha = 0.7143$, which implies that the labor share in production is close to 0.64, as in Prescott (1986). The capital depreciation rate is set to $\delta = 0.12$, as calculated using the Compustat data.

We calibration the serial correlation and standard deviation of the firm-specific shocks, together with adjustment cost parameters to jointly match the following moments: serial correlation of investment rate, standard deviation of investment rate, average investment rate, average acquisition capital ratio, and fraction of firms purchasing capital from the acquisition market.[8] The resulted serial correlation coefficient and standard deviation for firm-specific shocks are respectively 0.891 and 0.187. The standard deviation is much smaller than the estimates based on plant level data in Cooper and Haltiwanger (2006). The parameters of adjustment costs are $\gamma_n = 0.271$, $\gamma_u = 0.167$, and $\Phi = 0.012$. The value of γ_n is within the reasonable range of values as estimated in Cooper and Haltiwanger (2006) and Hall (2004).

[8] In the earlier version of the paper, we used the serial correlation and standard deviation of firm-specific shocks estimated by Cooper and Haltiwanger (2006), where the authors use a sample of large U.S. plants, the Longitudinal Research Database (LRD) 1972-1988. Compustat data sample used in our paper spans from 1981 to 2004, and displays different values of investment moments from the LRD 1972-1988 data.

With the above serial correlation and standard deviation of shocks, we discretize the firm-specific shock process into 17 state grids and form a Markov transition probability following the method of Adda and Cooper (2003). The productivity signal q for entrants is drawn from a truncated exponential distribution, $\Omega(q) = \widehat{q}(1 - e^{-\lambda q})$, with $q \in [\underline{\varepsilon}, \overline{\varepsilon}]$ and $\widehat{q}$ the scale parameter that makes $\Omega(q)$ a cumulative distribution function. Here, $\underline{\varepsilon}$ and $\overline{\varepsilon}$ are respectively the minimum and maximum states of discretized shocks. We set $\lambda = 1.20$ to match the average productivity of entrants relative to that of incumbent firms (0.96), and set $\psi = 0.27$ to match the average employment size of entrants relative to that of incumbents (0.60). Both the relative productivity and relative employment size of entrants are documented in Lee and Mukoyama (2010) for the U.S. plants.

The probability of exit is set to $s = 0.055$, as in Lee and Mukoyama (2010), to match the exit rate of U.S. plants. We assume that 60 percent of firms that exit can choose to sell their capital in the acquisition market, leading to $\xi = 0.60$. From 1986 to 2004, an average of 4 percent of firms were de-listed from Compustat data set each year, of which close to 60 percent are the result of mergers and acquisitions.

[Table 1 here.]

6.2 Baseline results

Table 2 summarizes predictions of the baseline model, compared with the data moments. In this table, the aggregate investment rates are ratios of total investment to total capital stock, while the average investment rates are averages of firm-level investment divided by capital stock. The rate of investment in used capital is for positive value of reallocation, consistent with acquisition in Compustat. The aggregate output is measured as the total consumption plus total investment.

The results indicate that the model is fairly successful at matching firm-level moments. It predicts a higher investment in used capital than that in Compustat. It also predicts a ratio of 12 percent for the average investment in used capital to total investment–very close to that in Compustat. It is also consistent with the ratio of 12 percent documented by Eisfeldt and Rampini (2007), using micro data from the Annual Capital Expenditure Survey. For aggregate moments, the model generates numbers that are roughly in line with the aggregate data. About 25 percent of capital formation is through the acquisition of used capital from surviving and exited firms, a little larger than that in Compustat data. The aggregate investment in used capital is 4 percent of the aggregate capital stock, compared to less than 3 percent in the data. The model predicts that 21 percent of surviving firms participate in the acquisition market, fewer than the number in the data. The aggregate capital adjustment cost is 5 percent of total firm output, which is roughly consistent with the estimate in Cooper and Haltiwanger (2006). In addition, the capital-output ratio is 1.4, while the aggregate for the United States is 1.7.

It should be noted that the average share of investment in used capital in total investment is much smaller than the aggregate share of investment in used capital because close to 80 percent of firms have zero investment in used capital.

7 Reallocation and Productivity

Having calibrated the model, we now quantify the gains in labor productivity from the increased reallocation of capital among firms as the result of an exogenous shock to the adjustment cost for used capital. With decreasing returns to scale in the production function, it is not obvious that an increase in reallocation would necessarily improve aggregate productivity. On the one hand, capital flowing to more-productive (higher ε) firms makes them larger, and therefore aggregate productivity can increase because the weight of more-productive firms increases. On the other hand, the average productivity of capital may decline as a firm becomes larger, while the average labor productivity may increase if the firm substitutes capital for labor.

Let Y and L be aggregate output and aggregate labor input, respectively. The aggregate labor productivity is defined as

$$Y_l = \frac{Y}{L} = \frac{\int y(\varepsilon,k)d\mu(\varepsilon,k)}{\int l(\varepsilon,k)d\mu(\varepsilon,k)},$$

where $y(\varepsilon,k)$ is the firm's output and $l(\varepsilon,k)$ is the firm's labor input. The improvement in aggregate productivity can arise from changes in firm distribution and from changes in the output and labor input of individual firms.

An alternative and extensively used measure of aggregate productivity is the (weighted) average labor productivity, defined as

$$\overline{Y_l} = \int \frac{y(\varepsilon,k)}{l(\varepsilon,k)} d\mu(\varepsilon,k).$$

Baily et al. (1992) and Foster et al. (2001) use this average measure to decompose productivity growth into within-firm growth, between-firm growth, and contribution of entry and exit. In our model, after plugging the optimal labor choice into the expression for average productivity, we obtain the firm-level labor productivity as follows: $\frac{y}{l} = \frac{w}{\alpha\nu}$. The average labor productivity is then reduced to $\overline{Y_l} = \frac{w}{\alpha\nu} \int d\mu(\varepsilon, k)$. Gains in productivity from increased reallocation across firms are determined by the equilibrium wage rate and the firm distribution. The average labor productivity is always equalized in our model in equilibrium, since there are no frictions in the labor market. The within-firm and between-firm components in the decomposition by Baily et al. (1992) and Foster et al. (2001) are degenerate in our model to growth in the wage rate. The contribution of changes in firm distribution to average labor productivity arises from the increased number of firms and increased wage rate.

Before bringing our model to test in the data, we first conduct a counter factual experiment in which we shut down the used capital market entirely, this allows us to quantify how important reallocation is in the model economy. When capital reallocation is shut down due to high cost of reallocation, aggregate output decreases by 2.2 percent and aggregate labor productivity drops by 3 percent. The decrease in aggregate labor productivity arises because when capital reallocation is too costly, the demand for investment falls, leading firms to downsize and reduce their output. In addition, firms substitute labor for capital. Thus, the ability to reallocate capital through the used capital market helps to increase labor productivity and, in our model, this channel is quantitatively important.

7.1 Effects of reallocation cost in the 1980s

In this section, we examine to what extent increased capital reallocation contributed to growth in aggregate labor productivity in 1980s. The experiment is conducted as follows. We choose the two years (1984 and 1985) as the base year before the reallocation increased, and calculate the increase in reallocation from the 1984-1985 average value to the 1986-1990 average value. We reduce the adjustment cost of used capital in the model so that the resulted increase in the firm-level reallocation rate equals to that increase shown in the data. The participation cost in acquisition market is also reduced so that the increase in fraction of firms acquiring capital from used capital market matches that in the data. We then calculate the increase in aggregate labor productivity resulted from reduced cost of reallocation. This increased productivity in the model is compared to the increase in aggregate labor productivity from the

1985-1986 average value to the 1987-1991 average value. This will allow us to find how much productivity growth in the data can be attributed to increased reallocation. Note that we assume that the effect of increased reallocation on productivity has a one-year lag, for two reasons. First, reallocation can be a long process in reality, hence its effect may be lagged. Further, this assumption is consistent with the timing of reallocation in the model, i.e., firms purchase capital for production in one period later.[9]

In the mid-1980s, the United States experienced a wave of mergers and acquisitions. The annual total value of all acquisitions rose from 3.1 percent of GDP in 1984 to 4.8 percent in 1988 (see Holmstrom and Kaplan, 2001). Nearly half of all U.S. corporations received a takeover offer in this period. During this period, a large proportion of acquisitions were leveraged buyouts, with investment groups and managers borrowing to buy back firm shares. After this wave of acquisitions, some firms went private. The Compustat data for this period show that the average rate of investment in new capital declined and the average rate of investment in used capital increased, as shown in Table 3. Between 1984 and 1990, the average reallocation rate rose by 3.1 percentage points, from 7.3 percent in 1984-1985 to a 5-year average of 10.4 percent in 1986-1990. Investment rate for new capital fell by 4.8 percentage points.[10] Since many firms became private and were delisted from Compustat over this period, the increase in the reallocation rate is likely underestimated.[11] Accompanying the increased mergers and acquisitions is the productivity growth. From the period 1985-1986 to the period 1987-1991, aggregate labor productivity (ALP) increased by 8.6 percent, while aggregate capital productivity (ACP) rose by 1.6 percent.[12]

[Table 3 here.]

The intensified reallocation of capital in the late 1980s is attributed to stock prices that were low relative to the cost of building new firm capacity. Expanding by taking over the capital of other firms appeared less costly than building a new plant. Jensen (1993) takes the view that acquisitions in the 1980s were a reaction of capital market participation to corporate mismanagement of conglomerates.

[9] Assuming a two-year lag for the effect of increased reallocation gives similar results.

[10] Note that we use the average rates of firm-level investment and reallocation, not the aggregate rates.

[11] When plotting the firm-size distributions in 1985 and 1989, we find that the size distribution in capital shifted to the left between 1985 and 1989.

[12] Productivity measures are historical data from manufacturing multifactor productivity tables by the Bureau of Labor Statistics (BLS). Data can be downloaded at http://www.bls.gov/mfp/tables.htm. BLS productivity data use capital service instead of capital stock, but BLS assumes that capital service is proportional to capital stock in calculating capital productivity. Capital productivity in the model and in data is consistent since we look at the growth rate.

In addition, the U.S. antitrust law was less strictly enforced during this period, which may also have facilitated increased mergers and acquisitions (see Ravenscraft, 1987).

Mapping the model to these contributing factors suggests that the acquisition costs of used capital decreased, causing an increase in reallocation activity among firms. Other changes, such as the increase in aggregate TFP, do not appear to be important for increased reallocation. To quantify the effects of increased reallocation, we take 1984-1985 as the base year, when reallocation is low, and take the period of 1987-1991 as the years when reallocation is high. The strategy is to reduce the cost of used-capital adjustment to the extent that the model would predicts an increase in the firm-level average rate of reallocation by 3.1 percentage points. Participation cost in used capital market is reduced in the same fashion so that the model would predicts an increase in the fraction of firms buying (not selling) capital in used capital market by 2.7 percentage points. Once the used capital adjustment cost and participation cost are reduced, gains in productivity are then calculated. The parameter values that meet these requirements are $\gamma_u = 0.699$ and $\Phi = 0.046$. With these values for the acquisition cost of used capital, the model predicts that more firms now participate in the acquisition market, and that firms already in this market acquire a higher fraction of used capital in total investment, as shown in Table 3. Following the cost decrease, the average rate of investment in used capital increases by 3.1 percentage points, matching the increase shown in the data. The fraction of firms buying capital from used capital market is slightly higher than that in the data. Meanwhile, the resulted average rate of investment in new capital falls by 0.3 percentage points.

Figure 3 shows how the policy function of firms shifts as the cost of investing in used capital falls. The horizontal axis represents the current capital stock and the vertical axis represents the current realization of productivity shock. Solid lines indicate areas where the two types of investment split for the baseline model, and dashed lines indicate the splits when the cost of used capital falls. When the cost decreases, more firms participate in the acquisition market; hence, investment in the acquisition market increases, and investment in new capital decreases.

[Figure 3 here.]

Table 3 shows that the aggregate effects of increased reallocation at the firm level are significant. Consistent with changes in the data, firm-level investment in new capital falls. Because capital becomes less costly to acquire, firms tend to increase their capital size and reduce their demand for labor. Indeed, the average labor demand of firms decreases by 0.85 percent. Therefore, the capital-labor ratio increases. With decreasing returns to scale in production technology, the output-capital ratio will decrease and the output-labor ratio will increase if capital size is larger and labor demand is less. Aggregate labor productivity increases by close to 1.7 percent, although the output-capital ratio falls by 1.2 percent. The BLS data show that over the 1987-1991 period, aggregate labor productivity increased by 8.6 percent from its 1985-1986 average value. Therefore, about 20 percent of the growth in labor productivity during the 1987-1991 period is attributable to the increased reallocation of capital across firms.

The sources of increased aggregate labor productivity are increased aggregate output and decreased employment. We use the logarithm form of productivity to ease its decomposition, $\Delta \ln Y_l = \Delta \ln Y - \Delta \ln L$. Increased output accounts for 89 percent of productivity growth, while decreased labor input accounts for 11 percent. The decreased cost of capital adjustment and of participation in the acquisition market make investment in used capital more appealing than investment in new capital, leading to increases in both used-capital investment and total investment. This has several consequences. First, consumption increases due to higher firm output and lower cost of capital, thus the measured aggregate output (consumption and investment in new capital) increases. Moreover, less-costly capital adjustment drives firms to substitute capital for labor, causing a decline in aggregate labor input. Second, changes in the demand for capital and labor induce price changes. Both the equilibrium wage rate and the relative price of used capital increase. The increased wage rate results from increased entry. When the adjustment cost falls, the expected value of entry tends to increase and, hence, more firms enter the industry. Competition among firms for labor input then raises the wage rate, which in turn tends to lower the expected value of entry. Figure 4 shows the cumulative distribution of capital size in the baseline case and in the experiment. In equilibrium, the expected value of entry does not change, so the free-entry condition still holds.

[Figure 4 here.]

At the firm level, average labor productivity increases by 1.3 percent relative to the baseline value. In logarithm form, growth in average labor productivity is given by

$$\Delta \ln \overline{Y_l} = \Delta \ln w + \Delta \ln \left(\int d\mu(\varepsilon, k) \right).$$

Using this decomposition, we find that 95 percent of the increase in firm-level productivity is accounted for by the increased equilibrium wage rate, and the rest 5 percent is accounted for by the change of firm distribution.

7.2 Reallocation and productivity in the 1990s

The same model can also help explain the growth in productivity during the merger wave of the late 1990s. In this section, we conduct a similar analysis of productivity growth due to increased reallocation. Investment rate and reallocation rate in the base year are calculated as the average values of 1994-1995, respectively. We examine how much the growth of aggregate labor productivity from the period 1995-1996 to the period 1997-2001 is attributable to the increase in reallocation from the period 1994-1995 average to the period 1996-2000 average.

The U.S. economy experienced extraordinarily fast growth in the 1990s, accompanied by a wave of mergers and acquisitions. The BLS data show that aggregate labor productivity rose by 18 percent between 1995-1996 and 1997-2001, as shown in Table 4, while aggregate capital productivity rose by merely 2 percent. During the same period, merger activities also increased. In 1999, the total value of assets being reallocated through mergers and acquisitions reached a historically high level, at 15 percent of the GDP. Table 4 shows that over the period 1996-2000, the average rate of investment in new capital increased by 0.3 percentage points from that in 1994-1995, while the average rate of investment in used capital rose by 7.9 percentage points.

[Table 4 here.]

To examine the productivity effects of increased capital reallocation, the same strategy is applied as in the previous experiment, and the adjustment cost for used capital is changed to that at which the model would generate an increase in average rate of used capital acquisition (from the baseline case) of 7.9 percentage points, the same magnitude displayed in the data. At the same time, participation cost is reduced so that in the model the fraction of firms buying (again not selling) capital from used capital market rises by 4.9 percentage points. The values for the cost parameters that meet these requirements are $\gamma_u = 0.4975$ and $c_f = 0.0454$. Table 4 shows that, with the reduced adjustment cost for used capital, aggregate labor productivity increases by 3.9 percent, accounting for 22 percent of its increase in the data. Aggregate capital productivity decreases by 2.6 percent from the baseline case. Finally, decomposition in logarithm form shows that close to 90 percent of labor productivity growth is due to increased output, and only 10 percent is due to the shrunk labor input.

Aggregate capital productivity in both experiments declines, a result of the increase in capital input with decreasing returns to scale in production function. This result appears to be inconsistent with the data because capital productivity increases in the data. With a more careful look at the data, we find that capital productivity growth is positive but very small in the data, and negative in some years during both merger waves. Changes in capital productivity in both the model and the data are very small, relative to growth in labor productivity.

8 Conclusions and future research

Given that some firms are more productive than others and some managers have better managerial ability than others, it would be most efficient to have all capital and labor organized for production in the most productive firms. In reality, both the production technology (returns to scale) and reallocation frictions prevent capital and labor from flowing from less-productive firms to more-productive firms. These frictions are not only informational and technical, but they can also be related to policy and institutions. The model in this paper is another step toward quantifying the benefits of reducing these frictions. Our analysis demonstrates that removing reallocation frictions boosts aggregate labor productivity. About 20 percent of the growth in aggregate labor productivity can be accounted for by increased capital reallocation. This implies that government policies that facilitate reallocation would boost productivity.

Our model assumes the existence of persistent productivity shocks and a perfectly competitive labor market. In equilibrium, all firms have the same labor productivity, which equals the equilibrium wage rate. However, empirical evidence shows that labor productivity at the firm and plant levels is not equal among establishments. This begs an extension to the current model to explain the dispersion of labor productivity across firms, and to investigate the contribution of worker reallocation to the growth of productivity. This can be done either by assuming that firms differ in their ability to use the same workers, or by introducing heterogeneous workers who have a different productivity or quality. At the aggregate level, a gain in productivity arises either from the increased share of firms using workers more efficiently or from increased worker quality.

A Data

The U.S. data on investment and reallocation are from Compustat 1975-2011. Before 1975, the variable on sales of capital was unavailable. We use the perpetual inventory method to obtain the capital stocks in which we take into account changes in capital due to acquisitions.

For investment, we use the Compustat item *CAPX*, expenditures on property, plant and equipment (PPE). This item excludes PPE of acquired firms. We use *SPPE* (funds received from the sales of PPE) as the sales of capital. This item can include both the sales and retirement of capital. Another item *PPEVR* is available to represent the retirement of capital, but discontinued after January 1997. For this reason, we do not differentiate between capital sales and capital retirement in capital accounting. For our purpose, investment in capital is defined as expenditures on PPE net the sales of PPE. We use the item *AQC* to represent the addition to capital stock from the PPE of acquired firms. But *AQC* not only includes acquired PPE but also can include other assets and long-term debt carried from the acquired firms. Where we believe that it covers more than the amount of acquired PPE, we adjust the values of *AQC*, by checking the consistency of the accounting of PPPE book values.

Capital stock. We follow the literature to calculate the replacement cost of capital stock using the perpetual inventory methods based on Salinger and Summers (1983). Let K_{ijt} be the capital stock of firm i in industry j at year t, and let I_{ijt} be the firm's investment in capital. The perpetual inventory method is as follows

$$K_{ij,t} = (1-\delta_j)\left(\frac{P_t}{P_{t-1}}K_{ij,t-1} + I_{ij,t}\right),$$

where δ_j is the depreciation rate of capital in industry j. The recursion is initiated using the firm's book value of PPE in the year of 1975. To calculate the depreciation rate of capital for the 2-digit SIC industries, we first obtain the useful life of capital at the firm level, denoted as

Table 5 reports the investment and reallocation moments for manufacturing industries. In this table, firm-level moments are computed as the averages of firm-level values, e.g., investment rate is the average of firm investment rates. Spike rate is the proportion of firm-year observations with an investment rate larger than 25 percent. Cooper and Haltiwanger (2006) define an investment spike if investment rate is larger than 20 percent. But in their plant-level sample, the average investment rate is 12 percent, smaller than the 17 percent of our average investment rate. Inaction is the proportion of firm/year observations with an investment rate less than 1 percent. In the case of reallocation (acquisition), firms not acquiring capital from other firms are considered to have zero acquisition and are not counted in calculating the firm-level reallocation rate. Aggregate moments are computed as the averages of annual aggregate moments. For example, the aggregate reallocation rate (acquisition capital ratio) in each year is calculated as the total value of acquired capital divided by the total value of capital in the entire industry. We then take the average of this ratio over the period of 1980-2005.

[Table 5 here.]

A.1 Sample selection

We use Compustat data item capx for capital investment,

B Proofs

B.1 Derivation of threshold value of investment rate $\tilde{i}$

The firm chooses $x_u = 0$ if

$$i+\frac{\gamma_n}{2}i^2 \le \min\{i-i_u+p_u i_u+\frac{\gamma_n}{2}(i-i_u)^2+\frac{\gamma_u}{2}i_u^2+\Phi, p_u i+\frac{\gamma_u}{2}i^2+\Phi\}.$$

Let $\tilde{i}$ be the total investment rate at which the firm is indifferent between $x_u = 0$ and $(x_n \neq 0, x_u \neq 0)$, then $\tilde{i} = \frac{1}{\gamma_n}(p_u - 1 \pm \sqrt{2\Phi(\gamma_n+\gamma_u)})$. That is, the firm chooses $x_u = 0$, investing only in new capital when

$$i \in \left(\frac{1}{\gamma_n}(p_u-1-\sqrt{2\Phi(\gamma_n+\gamma_u)}), \frac{1}{\gamma_n}(p_u-1+\sqrt{2\Phi(\gamma_n+\gamma_u)})\right). \quad \text{(B-1)}$$

Another threshold interval of total investment rate can be derived by comparing the cost of $(x_n = 0, x_u \neq 0)$ and the cost of $(x_n \neq 0, x_u = 0)$. If the firm chooses $x_u = 0$, then we have $p_u i + \frac{\gamma_u}{2}i^2 + \Phi > i + \frac{\gamma_n}{2}i^2$. Then, if

$$i \in \left(\frac{1}{\gamma_n-\gamma_u}(p_u-1-\sqrt{(p_u-1)^2+2\Phi(\gamma_n-\gamma_u)}), \frac{1}{\gamma_n-\gamma_u}(p_u-1+\sqrt{(p_u-1)^2+2\Phi(\gamma_n-\gamma_u)})\right), \quad \text{(B-2)}$$

the firm would choose $x_u = 0$ over $x_n = 0$.

As shown below in the proof of Proposition 1, the cost of choosing $(x_n = 0, x_u \neq 0)$ is always larger than the cost of choosing $(x_n \neq 0, x_u \neq 0)$, suggesting that on the right hand side of (B-1), $(p_u i + \frac{\gamma_u}{2} i^2 + \Phi)$ is redundant. Therefore, the threshold interval (B-2) is also redundant.

B.2 Proof of Proposition 1

Proof. Note that total investment rate is non-zero, meaning that $x_u \neq 0$ if $x_n = 0$. For the proposition to hold, it suffices to show that the total cost of choosing $x_n = 0$ is always higher than that of choosing $(x_n \neq 0, x_u \neq 0)$ or $(x_n \neq 0, x_u = 0)$,

$$p_u i + \frac{\gamma_u}{2} i^2 + \Phi > \max\{i - i_u + p_u i_u + \frac{\gamma_n}{2}(i - i_u)^2 + \frac{\gamma_u}{2} i_u^2 + \Phi, i + \frac{\gamma_n}{2} i^2\}.$$

First we show that the cost of choosing $x_n = 0$ is greater than the cost of choosing $(x_n \neq 0, x_u \neq 0)$. The difference between these two costs is

$$\begin{aligned} & p_u i + \frac{\gamma_u}{2} i^2 - [i - i_u + p_u i_u + \frac{\gamma_n}{2}(i - i_u)^2 + \frac{\gamma_u}{2} i_u^2] \\ = & \frac{1}{\gamma_n + \gamma_u}(\gamma_u i - 1 + p_u)^2 \geq 0, \end{aligned} \tag{B-3}$$

where in the second line the optimal $i_u = \frac{1}{\gamma_n + \gamma_u}(\gamma_n i + 1 - p_u)$ is used. This positive difference of investment costs between the two cases implies that, for any non-zero investment rate, the firm never chooses $x_n = 0$ over $(x_n \neq 0, x_u \neq 0)$. It should also be noted that (B-3) holds even when $p_u < 1$.

Next, we show that the firm does not choose $x_n = 0$ over $(x_n \neq 0, x_u = 0)$. For small investment rates located in the threshold interval (B-1), the firm chooses $(x_n \neq 0, x_u = 0)$ over $(x_n \neq 0, x_u \neq 0)$. By (B-3), choosing $(x_n \neq 0, x_u \neq 0)$ is always less costly than $x_n = 0$. Therefore, the firm does not choose $x_n = 0$ over $(x_n \neq 0, x_u = 0)$. For investment rates outside the threshold interval, the firm chooses $(x_n \neq 0, x_u \neq 0)$ over $(x_n \neq 0, x_u = 0)$ and $x_n = 0$. This suggests that the firm does not choose $x_n = 0$ when investment is non-zero. □

References

Adda, J. and R. W. Cooper (2003). *Dynamic Economics: Quantitative Methods and Applications*. Cambridge, MA.: The MIT Press.

Baily, M. N., C. Hulten, and D. Campbell (1992). Productivity dynamics in manufacturing plants. *Brookings Papers on Economic Activity. Microeconomics 1992*, 187–267.

Bartelsman, E. J. and M. Doms (2000). Understanding productivity: Lessons from longitudinal microdata. *Journal of Economic Literature 38*(3), 569–594.

Cooper, R. W. and J. C. Haltiwanger (2006). On the nature of capital adjustment costs. *Review of Economic Studies 73*(3), 611–633.

Eisfeldt, A. L. (2004). Endogenous liquidity in asset markets. *Journal of Finance 59*(1), 1–30.

Eisfeldt, A. L. and A. A. Rampini (2006). Capital reallocation and liquidity. *Journal of Monetary Economics 53*(3), 369–399.

Eisfeldt, A. L. and A. A. Rampini (2007). New or used? investment with credit constraints. *Journal of Monetary Economics 54*(8), 2656–2681.

Erard, B. and H. Schaller (2002). Acquisitions and investment. *Economica 69*(275), 391–413.

Foster, L., J. C. Haltiwanger, and C. J. Krizan (2001). Aggregate productivity growth. lessons from microeconomic evidence. In *New Developments in Productivity Analysis*, pp. 303–372. National Bureau of Economic Research, Inc.

Hall, R. E. (2004). Measuring factor adjustment costs. *The Quarterly Journal of Economics 119*(3), 899–927.

Hansen, G. D. (1985). Indivisible labor and the business cycle. *Journal of Monetary Economics 16*(3), 309–327.

Holmstrom, B. and S. N. Kaplan (2001). Corporate governance and merger activity in the united states: Making sense of the 1980s and 1990s. *Journal of Economic Perspectives 15*(2), 121–144.

Hopenhayn, H. and R. Rogerson (1993). Job turnover and policy evaluation: A general equilibrium analysis. *Journal of Political Economy 101*(5), 915–38.

House, C. L. and J. V. Leahy (2004). An ss model with adverse selection. *Journal of Political Economy 112*(3), 581–614.

Jensen, M. C. (1993). The modern industrial revolution, exit, and the failure of internal control systems. *Journal of Finance 48*(3), 831–80.

Jovanovic, B. and P. L. Rousseau (2002). The q-theory of mergers. *American Economic Review 92*(2), 198–204.

Jovanovic, B. and P. L. Rousseau (2004). Mergers as reallocation. Working paper, New York University.

Kaplan, S. N. (2000). Introduction. In S. N. Kaplan (Ed.), *Mergers and Productivity*. The University of Chicago Press.

Kurmann, A. and N. Petrosky-Nadeau (2007). Search frictions in physical capital markets as a propagation mechanism. Cahiers de recherche 0712, CIRPEE.

Lee, Y. (2005). The importance of reallocations in cyclical productivity and returns to scale: evidence from plant-level data. Working Paper 0509, Federal Reserve Bank of Cleveland.

Lee, Y. and T. Mukoyama (2010). Entry, exit and plant-level dynamics over the business cycle. Working paper, University of Virginia.

Lichtenberg, F. R. (1992). *Corporation Takeovers and Productivity*. Cambridge, MA.: MIT Press.

Lichtenberg, F. R. and D. Siegel (1989). The effect of control changes on the productivity of u.s. manufacturing plants. *Journal of Applied Corporate Finance*, 60–67.

Maksimovic, V. and G. Phillips (2001). The market for corporate assets: Who engages in mergers and asset sales and are there efficiency gains? *Journal of Finance 56*(6), 2019–2065.

McGuckin, R. H. and S. V. Nguyen (1995). On productivity and plant ownership change: New evidence from the longitudinal research database. *RAND Journal of Economics 26*(2), 257–276.

Prescott, E. C. (1986). Theory ahead of business cycle measurement. Staff Report 102, Federal Reserve Bank of Minneapolis.

Pulvino, T. C. (1998). Do asset fire sales exist? an empirical investigation of commercial aircraft transactions. *Journal of Finance 53*(3), 939–978.

Ramey, V. A. and M. D. Shapiro (2001). Displaced capital: A study of aerospace plant closings. *Journal of Political Economy 109*(5), 958–992.

Ravenscraft, D. J. (1987). The 1980s merger wave: an industrial organization perspective. In L. E. Browne and E. S. Rosengren (Eds.), *The Merger Boom*, pp. 17–51. The Federal Reserve Bank of Boston.

Ravenscraft, D. J. and F. M. Scherer (1989). The profitability of mergers. *International Journal of Industrial Organization 7*(1), 101–116.

Restuccia, D. and R. Rogerson (2008). Policy distortions and aggregate productivity with heterogeneous plants. *Review of Economic Dynamics 11*(4), 707–720.

Rogerson, R. (1988). Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics 21*(1), 3–16.

Salinger, M. A. and L. H. Summers (1983). Tax reform and corporate investment: A microeconomic simulation study. In M. Feldstein (Ed.), *Behavioral Simulation Methods in Tax Policy Analysis*. The University of Chicago Press.

Schoar, A. (2002). Effects of corporate diversification on productivity. *Journal of Finance 57*(6), 2379–2403.

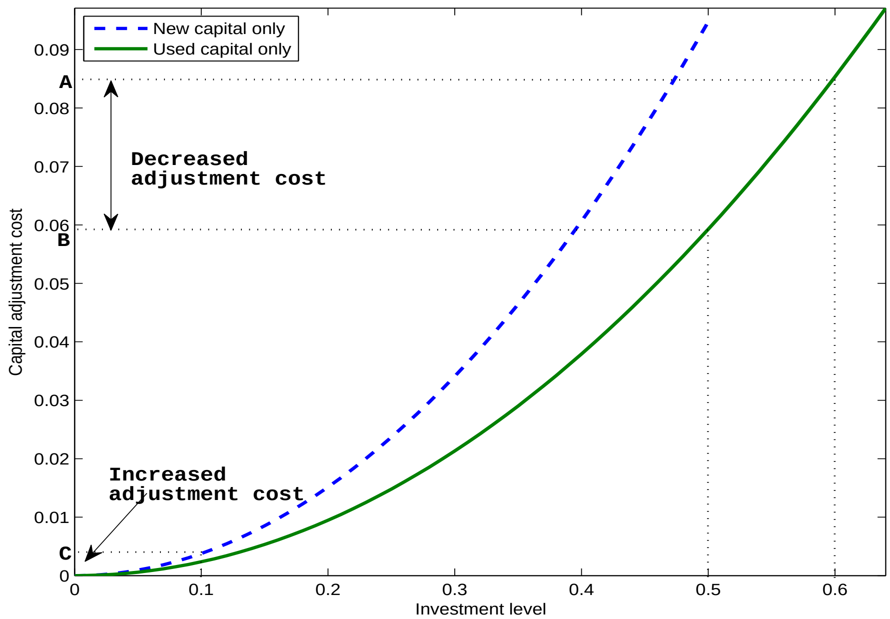

Figure 1: Illustration of Proposition 1.

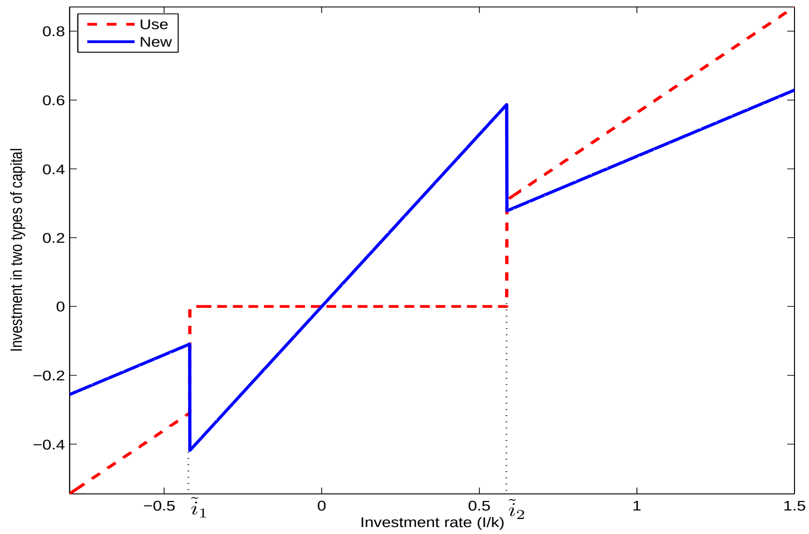

Figure 2: Optimal split between used and new capital

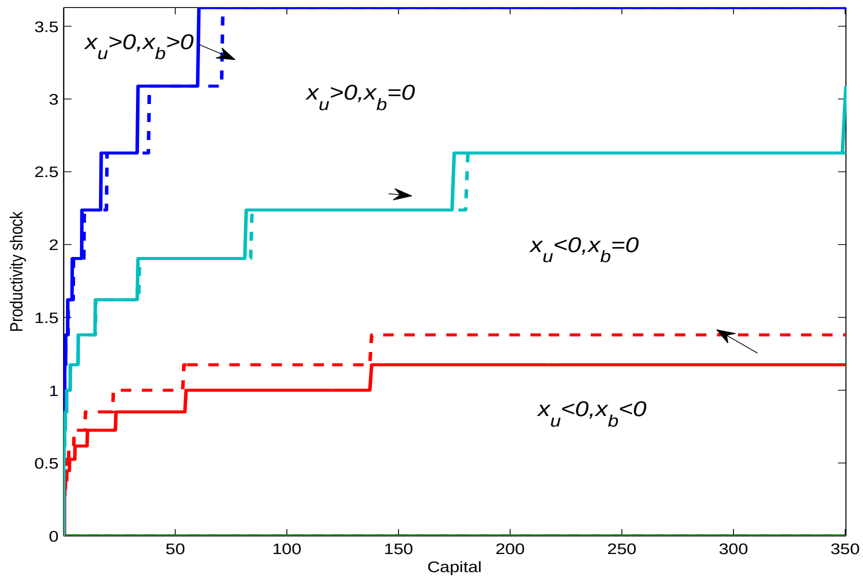

Figure 3: Changes in investment as the cost of used capital falls

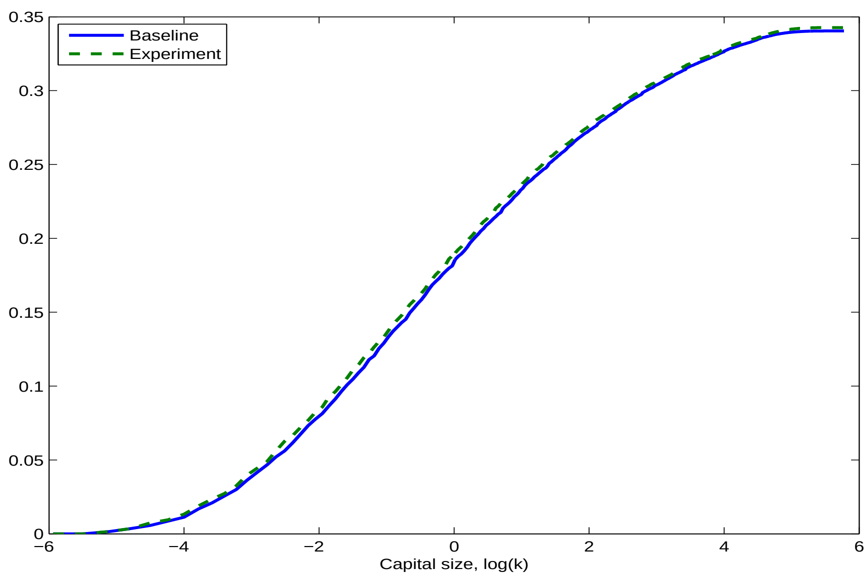

Figure 4: Cumulative distribution of capital

Table 1: Parameter value calibration

Parameter	Description	Value	Target	Model
β	discount factor	0.96	interest rate (4%)	4%
θ	labor disutility	0.97	labor force participation rate (0.80)	0.82
ν	production return to scale	0.896	Lee (2005)	
α	labor share in production	0.714	labor share in aggregate output (0.64)	0.62
δ	Capital depreciation rate	0.124	Compustat	
ρ_ε	serial correlation of shocks	0.45	serial correlation of investment rate (0.16)	0.17
σ_η	standard deviation of shocks	0.349	std. dev. of investment rate (0.24)	0.14
λ	distribution of signal for entrants	0.759	relative productivity shock of entrants (0.96)	0.90
ψ	fixed entry cost	0.225	relative employment size of entrants (0.6)	0.63
s	exit rate	0.055	Lee and Mukoyama (2010)	
ξ	exited firms sold in acquisition market	0.60	Compustat 1981-2004	0.60
γ_n	adjustment cost for new capital	0.475	average investment rate in new capital (0.16)	0.15
γ_u	adjustment cost for used capital	0.465	average investment rate in used capital (0.22)	0.25
Φ	participation in used capital market	0.006	fraction of firms investing in used capital (0.25)	0.27

Table 2: Baseline model moments compared with data moments

	Model	Data
Firm-level moments		
Average rate of investment in new capital	14.57%	15.65%
Average acquisition-capital ratio	25.42%	22.30%
Aggregate moments		
Aggregate rate of investment in new capital	11.77%	10.44%
Aggregate rate of investment in used capital	4.87%	4.32%
Share of investment in used capital in total investment	31.90%	25.70%
Participation in acquisition market	27.16%	25.40%
Relative productivity shock of entrants	0.90	0.96
Relative employment size of entrants	0.62	0.60
Aggregate output-capital ratio	0.82	-
Aggregate output-labor ratio	1.32	-

Table 3: Changes in reallocation and productivity growth: 1980s

	Average investment rate	Average reallocation rate	Participation rate	ACP	ALP	Aggregate output	Aggregate labor
	% point	% point	% point	%	%	%	%
Data	-2.68	2.25	2.32	1.58	8.60	9.40	0.72
Model	-0.00	2.24	2.33	-0.38	0.51	0.45	-0.07

Table 4: Changes in reallocation and productivity growth: 1990s

	Average investment rate	average reallocation rate	Participation rate	ACP	ALP	Aggregate output	Aggregate labor
	% point	% point	% point	%	%	%	%
Data	-0.82	3.07	5.64	2.00	18.02	16.96	-0.75 need to update data moments from latest do files
Model	0.11	3.09	5.68	-0.63	0.87	0.77	-0.09

Table 5: Investment and reallocation in Compustat 1981-2004

Moments	Firm-level	Aggregate
Average investment rate (%)	19.01	10.93
Fraction with investment rate > 25% (%)	23.79	-
Fraction with investment rate < -25% (%)	1.61	-
Fraction with investment rate >-1% and <1% (%)	3.65	-
Fraction with negative investment (%)	8.16	-
Serial correlation of investment rates	0.177	0.66
Standard deviation of investment rates	0.29	-
Average acquisition/capital ratio	21.54	3.50
Fraction of firms with acquisition	23.68	-

Table 6: Summary statistics of investment, compared with Cooper and Haltiwanger (2006)

Moments	Compustat 1975-1988	LRD 1972-1988
	percent	percent
Average investment rate	15.92	12.20
Fraction with investment rate > 20%	26.83	18.6
Fraction with investment rate < -20%	3.27	1.8
Fraction with investment rate >-1% and <1%	5.37	8.1
Fraction with negative investment	13.50	10.4
Serial correlation of investment rates	0.057	0.058
Standard deviation of investment rates	0.283	0.337