

Estimates of Seasonally-Adjusted Quarterly Total Factor Productivity for the Canadian Business Sector*

Shutao Cao[†]

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Abstract

In this note, we describe a new database of the seasonally-adjusted quarterly total factor productivity (TFP) for the Canadian business sector. The method we use extends the translog GDP function approach of estimating annual total factor productivity as in [Diewert and Yu \(2012\)](#).

Keyword: quarterly total factor productivity.

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[†]Economics Department, Trent University, Peterborough, Canada. Email: shutaocao@trentu.ca.

1 Introduction

Sound policy making and analysis requires timely data at a high frequency. Total factor productivity is one of major indicators on economic conditions, regularly examined in policy research. In practice, statistical agencies produce data of total factor productivity at the annual frequency, and often with a lag of one year or a longer period. Quarterly total factor productivity is desirable, as it is more frequent and more timely than annual data. However, quarterly measures of productivity are not available in most countries, with a few exceptions. [Fernald \(2012\)](#) develops quarterly total factor productivity for the United States business sector, and his measure adjusts for capacity utilization rates. [Cao and Kozicki \(2017\)](#) estimate quarterly total factor productivity for the Canadian business sector.

In this note, we develop a new database of growth accounting and total factor productivity at the quarterly frequency for the Canadian business sector. The new data differs from that by [Cao and Kozicki \(2017\)](#). First, the new productivity data is seasonally adjusted, while data developed by [Cao and Kozicki \(2017\)](#) is not. Seasonally-adjusted productivity measure is appealing because it can be directly used for policy and research. Further, policy analysis usually relies on economic data that is seasonally adjusted, thus the seasonally-adjusted measure is more consistent with other economic data than that developed in [Cao and Kozicki \(2017\)](#). The other limitation in Cao and Kozicki's quarterly measures is that some seasonally-unadjusted series are not available in the macroeconomic accounts of Canada, thus their seasonality has to be imputed. Another difference is that, in the new data described in this note, expected inflation of asset prices is taken into account in the user costs of capital.

The new seasonally-adjusted quarterly TFP, when annualized, matches closely the annual measure that is estimated using the same translog-GDP function approach. These measures growth at faster rates than the multifactor productivity by Statistics Canada.

2 Method

In this section, we review the method of estimating total factor productivity, and the definition of user costs of capital.

2.1 Translog GDP function approach

In estimating total factor productivity, we follow the translog function approach as in [Diewert and Yu \(2012\)](#) and [Kohli \(2004\)](#), among others, and see [Cao and Kozicki \(2017\)](#) for an overview. Let G^t be the current-price gross domestic output (GDP). Let y^t be the period t aggregate output in real value, and y_j^t be its j th component, where $j = 1, \dots, J, X, M$. J is the number of domestic outputs, X represents exports and M represents imports. Let P^t be the GDP deflator, and P_j^t be the deflator of the output j . By definition, $G^t = P^t \cdot y^t$. Two primary inputs are used for production: capital K^t and labour L^t . The growth of nominal GDP can be decomposed as follows:

$$G^t(P^t, K^t, L^t) / G^{t-1}(P^{t-1}, K^{t-1}, L^{t-1}) \approx A^t \gamma^t \beta^t. \quad (1)$$

The three components on the right hand side of the decomposition are given as follows. First, the growth of total factor productivity captures the change in gross domestic product assuming that the same amount of inputs are used for production and price level remains the same between two periods

$$\gamma^t = G^t(P, K, L) / G^{t-1}(P, K, L).$$

The growth factor of GDP due to changes in prices is given by

$$A^t = G^t(P^t, K, L) / G^t(P^{t-1}, K, L).$$

The growth factor of GDP due to changes in primary inputs is given by

$$\beta^t = G^t(P, K^t, L^t) / G^t(P, K^{t-1}, L^{t-1}).$$

If the GDP function has the translog form, the above decomposition identity holds exactly, as shown in [Diewert and Morrison \(1986\)](#). Moreover, A^t is then the Törnqvist index of the GDP deflator, $A^t = \frac{P^t}{P^{t-1}}$, and β^t is the Törnqvist index of input quantities, which is the product of Törnqvist indices of individual primary inputs, in this case, $\beta^t = \beta_K^t \cdot \beta_L^t$.

Decomposing the growth of real GDP is similar to the above case for nominal GDP. Let $g^t = G^t / P^t$ be the real GDP, with P^t the GDP deflator. The growth of real GDP is then decomposed

as follows

$$\frac{g^t}{g^{t-1}} = \gamma^t \cdot \beta_K^t \cdot \beta_L^t. \quad (2)$$

2.2 User cost of capital

In [Diewert and Yu \(2012\)](#), the user cost of capital for asset type i in period t is obtained as

$$U_i^t = (r^t + \tau_B^t + \tau_{P_i}^t + \delta_i^t) P_{K_i}^t.$$

In this definition, r^t is the after-tax real return of capital, to be solved by equalizing values of output and total inputs; τ_B^t is the business tax rate, common for all asset types; $\tau_{P_i}^t$ is the property tax rate, asset specific. Depreciation rate of capital, δ_i^t , can also vary with asset types.

The user cost of capital defined as above does not explicitly take into account capital gains, they are absorbed into the after-tax real return of capital.

In measuring total factor productivity for the United States, [Diewert and Fox \(2016\)](#) include capital gains (losses) explicitly in the user cost of capital. We extend their measure by incorporating business taxes. Let $\pi_i^t = P_i^{t+1}/P_i^t - 1$ be the inflation rate of price of asset type i . The *ex post* user cost of capital can be defined as

$$U_i^t = [1 + r^t + \tau_B^t + \tau_{P_i}^t - (1 + \pi_i^t)(1 - \delta_i^t)] P_{K_i}^t.$$

Equalizing the value of valued added and the value of total inputs, one can obtain the real after-tax rate of return to capital.

An alternative user cost is to use the expected inflation rates of asset prices, leading to a smoother user cost. Let $\bar{\pi}_i^t$ be the expected inflation of asset prices from period t to period $t+1$, the *ex ante* user cost of capital for asset type i can be defined as

$$\bar{U}_i^t = [1 + \bar{r}^t + \tau_B^t + \tau_{P_i}^t - (1 + \bar{\pi}_i^t)(1 - \delta_i^t)] P_{K_i}^t.$$

Different from [Diewert and Fox \(2016\)](#), we calculate $\bar{\pi}_i^t$ as a centered 24-quarter moving average of asset price changes. In the first 12 quarters, we simply use the 12-quarter average of price changes, and in the last 12 quarters, the centered moving averages are truncated such that in the last period expected inflation is the average of price changes over the last 12 quarters.

The real after-tax rate of return to capital can be obtained again by setting zero the difference between total value added and total inputs, both in current prices.

3 Data Sources

Data of inputs and final demands in business sector are built upon three main sources of data: Cansim tables by Statistics Canada, public-use micro files of the Labor Force Survey (LFS), and the estimates of annual total factor productivity. The last data is necessary as we cannot find some series at the quarterly frequency, thus we use the interpolated annual data series.

In creating the data series, where there are missing values, we use old archived series. Series of final demands and capital services can go back to 1961Q1, but the LFS data are not available before 1976. Therefore, our estimates of the quarterly TFP start with 1976Q1.

3.1 Final demands

Creating series of quarterly final demands is largely similar to that in the case of annual estimates. Shown by [Diewert and Yu \(2012\)](#), value added in the business sector can be expressed as

$$V_B^t = V_{CN}^t + V_I^t + V_X^t - V_M^t + (V_G^t - V_N^t).$$

V_{CN}^t is the final consumption at basic prices, $V_G^t - V_N^t$, the difference between government final consumption expenditure and the value added at basic prices in non-business sector, represents the sales from the business sector to the non-business sector less the purchases of intermediate inputs of the business sector from the non-business sector. We use data from Canadian System of Macroeconomic Accounts (CSMA) to measure components on the right hand side of the equation, allowing us to remove the residential rents from our estimation.

Private Consumption. We remove **residential rents**, both imputed from owner-occupied dwellings and paid rents, from household's total consumption expenditure. Starting with imputed rents. We obtain the quantities of imputed rents from Gross domestic product (GDP) at basic prices by industry (new Table 36-10-0434, formerly Cansim 379-0031), available for the period of 1997-2017, and we extend the series back to 1961 using the archived Gross domestic product (GDP) at basic prices by North American Industry Classification System (NAICS) (new Tables 36-10-

0398, formerly Cansim 379-0027) and by 1980 Standard Industrial Classification (SIC) (Table 36-10-0390, formerly Cansim 379-0007).

Current-price quarterly imputed rents at the basic price are not available in national accounts. We interpolate the annual implicit price of imputed rents which is created in estimating the annual total factor productivity. Current-price imputed rents are then obtained using the implicit price and quantity.

Series of paid rents are from Household final consumption expenditure (Table 361-00-107, formerly Cansim 380-0067). Total household consumption expenditure is obtained from the expenditure-based GDP, Table 36-10-0104 (formerly Cansim 380-0064).

We remove taxes from the consumption expenditure net of residential rents. Tax is calculated as Taxes on products minus subsidies on products and imports minus oil export tax minus Custom import duties, all obtained from Revenue, expenditure and budgetary balance - General governments.

Consumption of Government and NPISH. Both are obtained from the expenditure-based GDP (Table 36-10-0104).

Investment. Investment is obtained from the expenditure-based GDP table. It consists of private and government investments, and by NPISH. Private component in turn consists of investments in residential structure, non-residential structure, machinery and equipment, and intellectual property rights.

Changes in business inventory. As found in [Diewert and Yu \(2012\)](#), changes in inventory in the expenditure-based GDP table can have negative prices, which appears incorrect. We choose to use the current-price series of investment in inventories from the GDP table. We obtain implicit prices by interpolating the annual implicit prices, the latter is calculated from estimates by [Diewert and Yu \(2012\)](#) and the Industrial product price index.

Exports and imports. Series of exports and imports are from Exports and imports of goods and services, quarterly (NAPCS 2017 and NAPCS 2007). The dis-aggregated series are available back to 1961 only for the exports and imports of total goods and four categories of services. Thus, our measures of exports and imports are for those five categories. We cannot separate the

exports and imports by the government sector from those by the business sector, our measure of exports and imports may be over-estimated.

GDP at basic prices in business and non-business sectors. We cannot find current-price GDP at basic prices at the quarterly frequency for the business and non-business sectors. At the yearly frequency, we can measure V_B^t but do not use it in estimating total factor productivity, we would need to remove residential rents from it had we chosen to use it. To obtain V_N^t , we use series based on the input-output table. In the new Canadian System of Macroeconomic Accounts (CSMA), GDP at basic prices by industry in current prices is tabulated in Table 36-10-040 (formerly Cansim 379-0029), replacing Table 36-10-0395 (formerly Cansim 379-0024). In Table 361-10-040, educational and health services provided by government are not separately available. We instead use Gross domestic product (GDP) at basic prices, by sector and industry, provincial and territorial, Table 36-10-0487 (formerly Cansim 381-0030), where education and health services provided by government are available. Data are available from 2007 to 2014. We extend the series back to 1961 using data in Gross domestic product (GDP) at basic price in current dollars, System of National Accounts (SNA) benchmark values, special industry aggregations based on the North American Industry Classification System (NAICS).¹ We derive data after 2014 using quantities and prices of GDP at basic prices.

For quarterly Q_B and Q_N , we use GDP at basic prices by industry (monthly) and extend them back to 1961 using archived tables. We interpolate the annual implicit prices of GDP by sector to obtain the quarterly implicit prices. The current-price quarterly GDP by sectors is then the product of implicit prices and quantities.

3.2 Labor inputs

We follow [Diewert and Yu \(2012\)](#) to measure series of labor inputs for 36 types of workers: two by gender, three by age, three by education, and two by employment class (employee or self-employed). Quarterly labor inputs are obtained by interpolating the annual series using the Denton method, where the quarterly variation follows the quarterly series obtained from the public-use micro files of the Labor Force Survey (LFS).² The annual series of labor inputs

¹We found that Table 36-10-0395 (formerly Cansim 379-0024) have the same values of non-business GDP at basic prices as Table 36-10-0408 (formerly 381-0015), the latter is replaced by Gross domestic product (GDP) at basic prices, by sector and industry, provincial and territorial (Table 36-10-0487,formerly 381-0030).

²We implement this using denton command in Stata.

are based on Statistics Canada table, Hours worked and labour compensation by type of worker and industry (Table 36-10-0209, formerly Cansim 383-0024). The data are available up to 2010. For years after 2010, we use the LFS data.

Two issues need to be addressed. First, labor compensation is not available for the self-employed, we simply assume that it is the same as that for employees with the same educational attainment, age, and gender. Second, compensation is not available before 1997Q1 in the LFS. For the quarterly variation of compensation before 1997Q1, we use data from Wages, salaries and employers' social contributions. Thus labor compensation before 1997Q1 varies in the same way for all types of workers.

3.3 Capital stocks

Current-price series and quantities of capital stocks are obtained with the perpetual method. We use the capital stocks for 2012, obtained in the annual TFP estimation, as the beginning (in 2012Q1) capital stocks. For each asset i , we then apply the perpetual method to calculate capital stocks before and after 2012Q1,

$$K_i^{t+1} = (1 - \delta_i^t)K_i^t + I_i^t.$$

In the annual TFP estimation, we measure stocks and prices of 16 types of reproductive assets, they are

- Non-residential buildings.
- Engineering construction.
- Textile products, clothing, and products of leather and similar materials.
- Wood products.
- Plastic and rubber products.
- Non-metallic mineral products.
- Fabricated metallic products.
- Industrial machinery.
- Computer and electronic products.
- Electrical equipment, appliances and components.

- Transportation equipment.
- Furniture and related products.
- Other manufactured products and custom work.
- Mineral exploration and evaluation.
- Research and development.
- Software.

Stocks and flows are obtained from Investment in fixed non-residential capital by sector of industry and type of asset (Table 36-10-0097, formerly Cansim 031-0006). We follow [Diewert and Yu \(2012\)](#) to use the perpetual method to obtain real values of capital stocks for each type of assets.

At the quarterly frequency, we apply the law of motion for each asset by using quarterly investment. Unfortunately, the quarterly investment data, Gross fixed capital formation, are different from the annual data in Investment in fixed non-residential capital by sector of industry and type of asset. Not only the asset types are different, data for the same asset type can be different between the two data sources. This forces us to have the following asset types for quarterly capital stocks for which data are close between two sources:

- Non-residential buildings.
- Engineering construction.
- Machinery and equipment.
- Mineral exploration and evaluation.
- Research and development.
- Software.

The quarterly capital services, when annualized, grow at a slightly slower pace than the annual estimates, possibly due to that quarterly investment starts to depreciate after one quarter in the quarterly estimates while it depreciates after one year in the annual estimates.

Stock of business inventories. It is obtained from the National Balance Sheets Accounts and the implicit prices of investment in business inventories.

Lands. There are two types of land: agricultural land and non-agricultural business land. We interpolate the annual estimates of lands into quarterly frequency. The current-price value of total and business lands is from the National Balance Sheets Accounts, Table 36-10-0487 for the period of 1990-2017, and Tables 36-10-0510 and 36-10-0511 for the period of 1961-1996. The current-price value of agricultural lands is from Balance sheet of the agricultural sector (Table 32-10-0056, formerly CANSIM 002-0020), available from 1981 to 2017, which we extend back to 1961 using series from Table 32-10-0050 (formerly CANSIM 002-0007).

We have no price of business land, we thus follow [Diewert and Yu \(2012\)](#) to assume that the quantity of business lands does not change over time.

The quantity of agricultural lands is estimated from two tables: Value of farm capital at July 1, and Value per acre of farm land and buildings at July 1.

4 Results and comparison

Figure 1 shows growth rates of the annual TFP and the annualized quarterly TFP. The annualized TFP is obtained by taking the quarterly average within each year. The two growth rates are close, with a correlation coefficient of 0.99. The differences between the two series lie in primarily capital services. Annualized capital services grow at a slower average rate than annual capital services. Several sources contribute to the difference. The user cost of capital can be different due to different rates of asset price inflation used in quarterly and annual measures. The law of motion of capital at the quarterly and annual frequencies may give different levels of capital stocks. In addition, investment prices are different between the quarterly and the annual data at Statistics Canada, contributing to differences in real capital stocks.

Statistics Canada estimates the multifactor productivity (MFP) for the business sector, which has a much slower average growth rate than the annual TFP by the translog-GDP function approach, as shown in Figure 2.³ Such a difference is mainly attributed to a faster growth of capital services in the data by Statistics Canada. Capital services grow at an average annual rate of 3.3 percent in our measure, while it is 4.2 percent in Statistics Canada's data. Another difference lies in that in our measure of TFP, we exclude housing rents from GDP.

³Growth rates of annual TFP and Statistics Canada's MFP are highly correlated, with a coefficient of 0.92.

Capacity utilization. In our measure, quarterly TFP can be mis-measured as we do not take into account capacity utilization rates. Our TFP measure can represent both technology and capacity utilization. To remove capacity utilization from our measure of TFP, we follow [Cao and Kozicki \(2017\)](#) to adjust the measured quarterly TFP for utilization. We use the capacity utilization rate provided by Statistics Canada, which is defined as the ratio of actual output over potential output. The utilization-adjusted TFP growth equals the quarterly TFP growth minus percentage changes in utilization rates. [Figure 3](#) compares growth rates. The average growth rate of the adjusted TFP is slower than the original TFP growth by 0.05 percentage points, and is more volatile than the later.

5 Concluding remarks

We have constructed a database of seasonally-adjusted quarterly TFP for the Canadian business sector. Our measure has advantages of being more timely and frequent. When annualized, our measure matches closely with the annual TFP measure estimated based on the same translog-GDP function approach. The quarterly TFP is based on primarily publicly available data for the business sector, it can be updated once data of macroeconomic accounts are released.

The quarterly TFP should be used with caution, as its changes may also reflect variation in factors other than technology. For example, prices of exports and imports can change significantly from one quarter to another, some of which arise from the demand side. These price changes may lead to changes in both output and inputs, thus affecting the measured TFP.

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Figure 1: Comparing TFP measures, growth rates

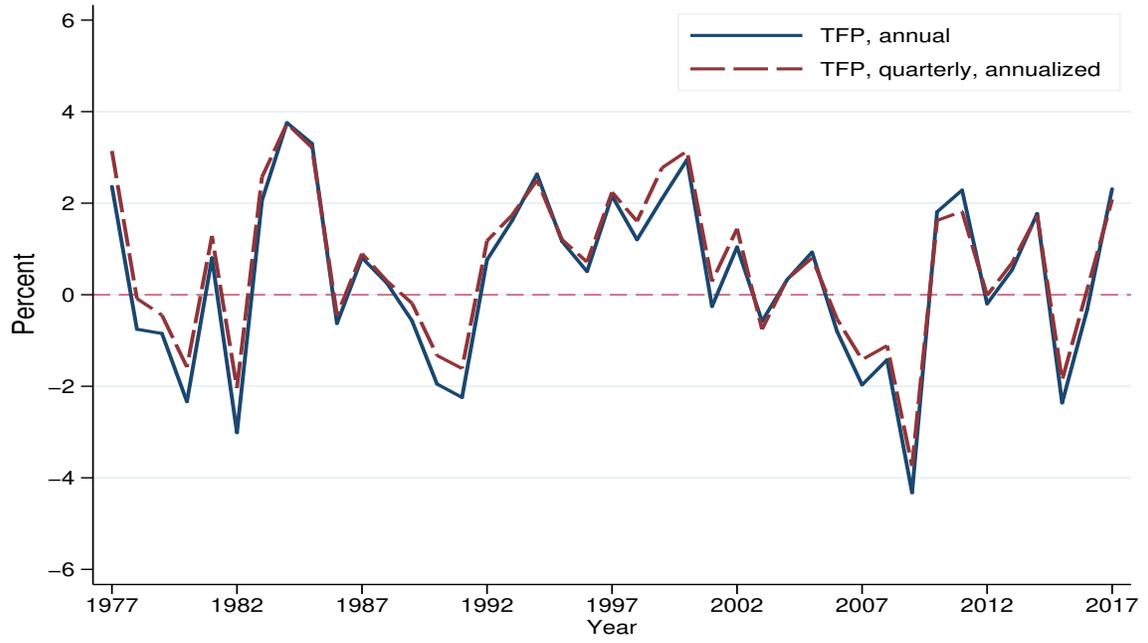


Figure 2: Comparing TFP measures, growth rates

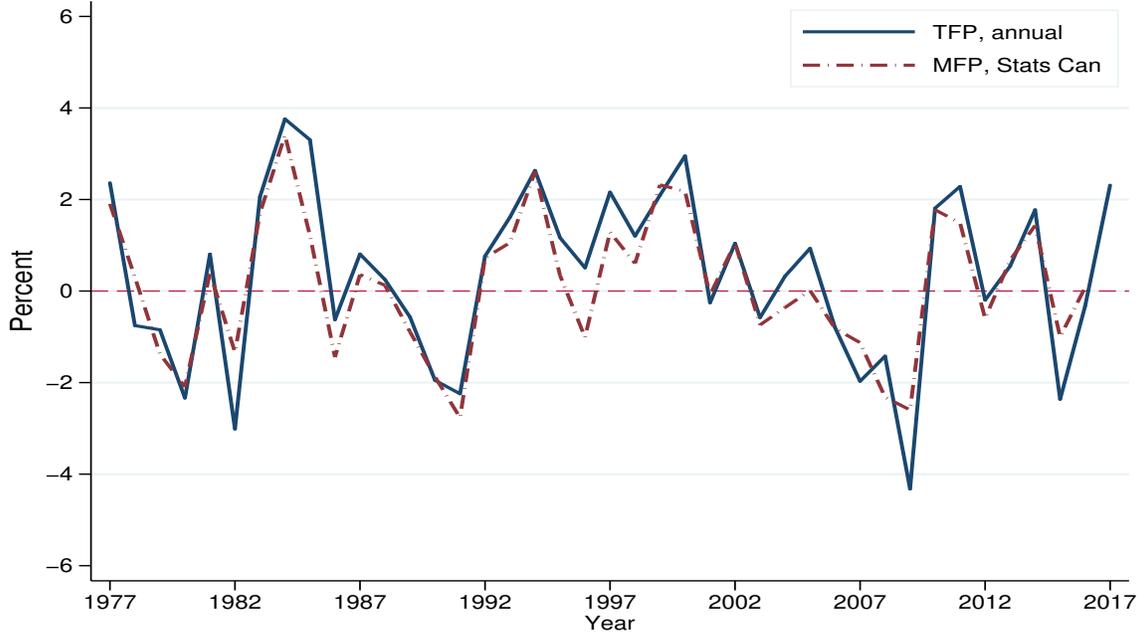


Figure 3: TFP growth, 4 quarter change in percentage

